Digital Platforms: Does Promoting Competitors Promote Competition? Online Appendix

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Abstract

This documents gathers the extensions of the main paper

OA.1 Omitted proofs

OA.1.1 Alternative benchmarks: low cost of a second visit

For formal simplicity and to fix ideas, in the body of the paper we chose to focus on the case when, absent a go-between, R-users would single home. We believe this assumption is realistic in the media sector, where reading a newspaper may require a lot of time, and readers have been shown to be very loyal to only a few sources of information (see Flaxman et al., 2016). Still, in practice some readers may multi-home. In markets other than media, such as flight search engine, many users may multi-home, owing to the relative user-friendliness of their interfaces, that makes multi-homing almost effortless. Here we consider alternative benchmarks for R-user participation and allow for any nonegative visit cost s. We show that as long as multi-homing is only partial, all qualitative results are maintained.

Assume first that $s \in [0 : \underline{\underline{s}}]$. In that case we showed in Proposition 1 that all R-users multi-home in the absence of interplatform references. Introducing references has two effects:

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- they allow all R-users to save the visit cost s
- they increase equilibrium quality by $k\frac{f+F}{4ct_r}$

Hence, following the introduction of references, user surplus is increased by s, on top on a positive quality effect when reference fees are positive.

When $s \in [\underline{\underline{s}} : \underline{s}]$, R-users multi-home in the absence interplatform references. The introduction of these references has two effects:

- they allow all R-users to save the visit cost s
- they increase equilibrium quality by $\frac{\alpha_a K - k(\alpha_a - f - F)}{4ct_r} - t_r - \frac{s}{k}$

Note that the cost savings on visits are same as when $s \in [0 : \underline{\underline{s}}]$. However too small fees induce a decrease in quality.

When $s \in [\underline{s} : \bar{s}]$, a measure $m(s)$ of R-users multi-home in the absence of a interplatform references. Hence references have three effects:

- they allow $m(s)$ R-users to save the visit cost s
- they decrease equilibrium quality by $\frac{k(2\alpha_a - f - F)}{4ct_r}$
- they allow $1 - m(s)$ users to view the k items of their platforms of second choice

The case in which $s \in [\bar{\bar{s}} : \infty)$ (absent a go-between, all users single-home) is treated in the body of the paper. If $s \in [\bar{s}, \bar{\bar{s}}]$ we showed in Appendix A that there are two pure Nash equilibira. They correspond to the case when $s \in [\underline{s} : \bar{s}]$ and the one covered in the body of the paper, respectively.

Figure 1 summarizes graphically the results of Proposition 2, 4, 5 and 6, when we allow for any nonnegative visit cost. It is immediate that Propositions 3 and 7, relative to the strategies of the go-betweens, are maintained.

[Figure 1 about here.]

OA.1.2 Proof of Lemma 1

Lemma 1 (perfect information):

Lemma 1 (Feasible set with full information) *Assume platforms have full information and are perfectly rational. The feasible set $\mathscr{F}$ is nonempty and bounded.*

Proof. We first allow for both platforms to be perfectly rational. We assume that the go-between sells a unidirectional sponsoring service, i.e., allows for a situation where platform j would sponsor content in platform i but not the other way around. We look for feasible fees f and F such that unilateral sponsoring is accepted by both parties. We assume that s is large enough so that there is no multi-homing.

We saw in the main text that all A-users multi-home and all their surplus is extracted by platforms, as shown in Armstrong and Wright (2007). This means we always have that $\gamma_i = \alpha_a$. When only j sponsors content in i, R-user utility is :

$$U_r^i(x,k) = \bar{u}_r + (q_i - t_r \mid x - x_i \mid)K + (q_j - t_r \mid x - x_j \mid)\, k \tag{OA.1}$$

$$U_r^j(x,k) = \bar{u}_r + (q_j - t_r \mid x - x_j \mid)K \tag{OA.2}$$

From these utility functions we derive the demand function:

$$n_r^i(k) = \frac{K-k}{2K-k} + \frac{(q_i - q_j)K + q_j k}{t_r(2K-k)} \tag{OA.3}$$

The platforms' profit functions are:

$$\begin{aligned}
\Pi_i(q_i) =& \alpha_a K n_r^i + f k n_r^i - c q_i^2 \\
\Pi_j(q_j) =& \alpha_a (K n_r^j + k n_r^i) - F k n_r^i - c q_j^2\,,
\end{aligned}$$

from which we derive that:

$$
\begin{aligned}
2cq_i =& (\alpha_a K + fk)\frac{\partial n_r^i}{\partial q_i} \\
=& (\alpha_a K + fk)\frac{K}{t_r(2K-k)} \\
2cq_j =& \alpha_a K \frac{\partial n_r^j}{\partial q_j} + k(\alpha_a - F)\frac{\partial n_r^i}{\partial q_j} \\
=& (\alpha_a K - k(\alpha_a - F))\frac{K-k}{t_r(2K-k)}
\end{aligned}
$$

It reduces to :

$$
q_i(k,f) = q^* \left(\frac{(1 + \frac{fk}{\alpha_a})K}{K - \frac{k}{2}} \right) \tag{OA.4}
$$

$$
q_i(k,F) = q^* \left(\frac{\left((1 - k\left(1 - \frac{F}{\alpha_a}\right)\right)(K-k)}{K - \frac{k}{2}} \right) \tag{OA.5}
$$

We can compute the equilibrium profits when there are k references:

$$
\Pi_i^*(k,f,F) = \Pi_i^*(0) + \alpha_a K(n_r^i(k) - n_r^i(0)) + kfn_r^i(k) + c(q_i^2(0) - q_i^2(k))
$$

$$
\Pi_j^*(k,f,F) = \Pi_j^*(0) - \alpha_a K(n_r^i(k) - n_r^i(0)) + k(\alpha_a - F)n_r^i(k) + c(q_j^2(0) - q_j^2(k))
$$

Define $P(k,f,F) \equiv \Pi_i^*(k,f,F) - \Pi_i^*(0)$ and $S(k,f,F) \equiv \Pi_j^*(k,f,F) - \Pi_j^*(0)$. The feasible set is all pairs (f,F) such that $P(k,f,F) \geq 0$, $P(k,f,F) \geq 0$ and $f \leq F$. We can verify that $f = F = -\alpha_a$ satisfies all three conditions, thereby proving the non-emptiness of $\mathscr{F}$ (Lemma 1):

$$
P(k, -\alpha_a, -\alpha_a) = \alpha_a K(n_r^i(k) - n_r^i(0)) + kfn_r^i(k) + c(q_i^2(0) - q_i^2(k,f)) \geq 0
$$

The inequality follows from the fact that for k small enough, $(K-k)n_r^i(k) - \frac{K}{2} \geq 0$,

$q_i^2(k, -\alpha_a) < q^*$. Similarly, we can show that $S(k, -\alpha_a, -\alpha_a) \geq 0$. Further, we have that

$$\begin{aligned}\frac{\partial P(k,f,F)}{\partial f} =& k n_r^i(k) > 0\\ \frac{\partial S(k,f,F)}{\partial F} =& -k n_r^i(k) < 0\end{aligned}$$

Given that taking f arbitrarily low results in $P(k, f, F) < 0$ and F arbitrarily high results in $S(k, f, F) < 0$, we conclude that $\mathscr{F}$ is bounded. This completes the proof of Lemma 1.

$P(k, f, F) \geq 0$ and $S(k, f, F) \geq 0$ translate into:

$$f \geq \frac{c}{k n_r^i(k)}(q_i^2(k,f) - q_i^2(0)) - \frac{\alpha_a K}{k}\left(1 - \frac{n_r^i(0)}{n_r^i(k)}\right) \tag{OA.6}$$

$$F \leq \frac{c}{k n_r^i(k)}(q_j^2(0) - q_j^2(k,F)) - \frac{\alpha_a K}{k}\left(1 - \frac{n_r^i(0)}{n_r^i(k)}\right) + \alpha_a \tag{OA.7}$$

To prove Lemma 1, we note that $f = F = -\alpha_a$ is in the feasible set: First, it obviously meets the condition that the go-between makes nonnegative profits. Second, relation (OA.4) shows we have $q_i(k, f) < q^*$. Developing the demand function (OA.3) when k is small results in condition (OA.6) being met. Finally, We carry out similar calculations to show that OA.7 holds.

Unfortunately, $n_r^i(k)$ depends on both f and F through $q_j(k, f)$ and $q_j(k, F)$. This means that when platforms rationally anticipate the impact of the fees applied to the competitor on its quality, the feasibility set $\mathscr{F}$ does not have an easy formulation. In order to have a simple formulation of the feasible set $\mathscr{F}$, we slightly relax the assumption of perfect information in Lemma 2. ■

Feasible set with partial information: Lemma 2 provides an explicit definition of the feasible set when platforms do not observe the fees faced by their competitor.

Lemma 2 (Feasible set with partial information) *Assume platforms have a prior* *over the fees faced by the other platform. There exist boundaries* $(\underline{f}, \bar{F}) \in \mathbb{R}^2$ *such that*

$$\mathscr{F} = \{(f, F) \in \mathbb{R}^2 / f \leq F, f \geq \underline{f}, F \leq \bar{F}\}$$

Proof. In practice, platforms see only the fees they are offered, but not the fees faced by their competitor. Assume that i and j form belief about the fee faced by the other platform, respectively F_e and f_e. Platforms accept to publish and promote, respectively, if fees f and F are such that $P(k, f, F_e) \geq 0$ and $S(k, f_e, F) \geq 0$. This translates into:

$$f \geq \underline{f} \equiv \frac{c}{k n_r^i(k)} (q_i^2(k, f) - q_i^2(0)) - \frac{\alpha_a K}{k} \left(1 - \frac{n_r^i(0)}{n_r^i(k, f, F_e)}\right) \tag{OA.8}$$

$$F \leq \bar{F} \equiv \frac{c}{k n_r^i(k)} (q_j^2(0) - q_j^2(k, F)) - \frac{\alpha_a K}{k} \left(1 - \frac{n_r^i(0)}{n_r^i(k, f_e, F)}\right) + \alpha_a \tag{OA.9}$$

The feasible set $\mathscr{F}$ is the set of all fees such that $F \leq \bar{F}$, $f \geq \underline{f}$ and $f \leq F$. Numerical applications show that this set is non empty for all expected fees smaller than α_a. In particular this is true when platforms make rational expectations about the fees set by a profit-maximizing go-between. ■

If firms are naive, and assume user choice and equilibrium quality are unaffected by references, (OA.6) and (OA.7) show that the feasible set is a triangle with $\underline{f} = 0$ and $\bar{F} = \alpha_a$. If firms foresee the change in users' choice but neglect the quality effect, (OA.6) and (OA.7) show that the feasible set has same size as the naive one, but $\underline{f}$ and $\bar{F}$ are both shifted to the left. This is due to the fact references allow i to capture more anchored users, at the expense of j.

OA.1.3 Proof of Proposition 4

We seek to analyze relation (13) in the case of general cost functions $C(q)$. We have that $\Delta U_r^i(x,0)=0$. We now look at positive deviations of k above 0:

$$\frac{\partial \Delta U_r^i(x,k)}{\partial k} = K\frac{\partial q_i}{\partial k} + q_i + k\frac{\partial q_i}{\partial k} - t_r(1-x) \tag{OA.10}$$

From (24), we have that $q_i = (C')^{-1}\left(\frac{\alpha_a K - k(\alpha_a-(f+F))}{2t_r}\right)$. $C'(q)$ is increasing in q, hence so is its inverse function. Hence if $\alpha_a > (f+F)$ the first term and two last terms in (OA.10) are negative. This means only the "diversity" effect is positive, whereas the quality effect is always negative. Using the implicit function theorem in (24), we derive that

$$\frac{\partial q_i}{\partial k} = -\frac{\alpha_a-(f+F)}{2t_r\frac{\partial^2 C}{\partial q^2}(q)} = -\frac{\frac{\partial C}{\partial q}(q^*)}{\frac{\partial^2 C}{\partial q^2}(q)}\frac{\alpha_a-(f+F)}{K\alpha_a}$$

Hence, these remarks result in:

$$\frac{\partial \Delta U_r^i(x,k)}{\partial k} = q_i - \frac{C'(q^*)}{C''(q_i)}\frac{\alpha_a-(f+F)}{\alpha_a}(1+k/K) - t_r(1-x) \tag{OA.11}$$

The convexity of the cost function ensures that $\frac{C'(q^*)}{C''(q^*)}>0$. When $u\equiv f+F$ is arbitrarily close to α_a, Assumption A1 ensures that references are always surplus maximizing: $\frac{\partial \Delta U_r^i(x,k)}{\partial k}>0$. Indeed, quality is restored to or above the no-reference level (see equation 9), and users enjoy more diversity of content.

Conversely, when $u=f+F=0$, a sufficient condition for references to decrease user surplus when k is small is that $C''(q^*)\leq\frac{C'(q^*)}{q^*}$ (i.e. the cost function is not too convex[1]). This condition is met if $C'(0)\geq 0$ and $C'''(q)<0$.

To sum up, when costs are not too convex and denoting $u=f+F$ and $g(u)\equiv \frac{\partial \Delta U_r^i(x,k,u)}{\partial k}$, we have shown that $g(0)<0$, $g(\alpha_a)>0$. $g(u)$ is monotonically increasing

[1] With costs functions of the form $C(q)=cq^{\gamma}$, this translates into $\gamma\leq 2$.

in u. Hence, by the intermediate value theorem, we have proved that there exists a unique $u^* \in (0, \alpha_a)$ such that references increase user-surplus if and only if $u > u^*$. Simple calculations show that $C(q) = cq^2$ translates into $u^* = \frac{4ct_r^2 + \alpha_a k}{K+k}$. This proves Proposition 4.

OA.1.4 Proof of Proposition 5

We use the expression of profits (8) to derive:

$$\frac{\partial \Pi_i}{\partial k} = \frac{1}{2}\left(\frac{\alpha_r}{t_r}(\alpha_a - f - F) + f - F\right) + c\frac{\alpha_a - f - F}{(4ct_r)^2}\left(\alpha_a(K-k) + k(f+F)\right)$$

For low fees $f = F < \frac{\alpha_a}{2}$ profits always increase with references. It is also easy to show that:

$$\begin{aligned}
\frac{\partial \Pi_i}{\partial F} &= -\frac{k}{2}\left(\frac{\alpha_r}{t_r} + 1 + \frac{\alpha_a(K-k) + k(f+F)}{4ct_r^2}\right) < 0 \\
\frac{\partial \Pi_i}{\partial f} &= -\frac{k}{2}\left(\frac{\alpha_r}{t_r} - 1 + \frac{\alpha_a(K-k) + k(f+F)}{4ct_r^2}\right) < 0\,,
\end{aligned}$$

where the second inequality requires assumption A1 to be verified. Hence low fees increase profits and therefore facilitate entry. The analysis of cross derivatives shows that the positive effect of references on entry is magnified by low fees:

$$\begin{aligned}
\frac{\partial^2 \Pi_i}{\partial F \partial k} &= -\frac{1}{2}\left(\frac{\alpha_r}{t_r} + 1 + \frac{\alpha_a K - 2k(\alpha_a - f - F)}{4ct_r^2}\right) \\
\frac{\partial^2 \Pi_i}{\partial f \partial k} &= -\frac{1}{2}\left(\frac{\alpha_r}{t_r} - 1 + \frac{\alpha_a K - 2k(\alpha_a - f - F)}{4ct_r^2}\right)\,,
\end{aligned}$$

are both negative for k small enough. The case of general cost functions $C(q)$ follows the same steps. It first notes that $\frac{\partial \Pi_i}{\partial F} = -\frac{k}{2} - C'(q)\frac{\partial q}{\partial F}$. Using the implicit function theorem in (24), it results that this expression is always negative. This means that platforms jointly choose a low sponsor fee, which is constrained by the participation constraint of the go-between: $F = f$. Under this constraint, $\frac{\partial \Pi_i}{\partial f} = -2C'(q)\frac{\partial q}{\partial F}$, which again is negative. Hence, bilateral negotiation between platforms results in setting both the sponsor and publisher fees at the minimum feasible level.

OA.1.5 Proof of Proposition 6

Denote $\Delta W(k, f, F) \equiv W(k) - W(0)$ the increase in social welfare due to references. It follows that:

$$\frac{\Delta W(k,f,F)}{k} = \overbrace{\alpha_a}^{\text{more views}} - \overbrace{\frac{3t_r}{4}}^{\text{average extra distance}} - \overbrace{\frac{k}{K}}^{\text{quality variation}} q^* + \overbrace{(f+F)\frac{K+k}{4ct_r}}^{\text{procompetitive fees}} + \overbrace{2c\frac{\alpha_a - (f+F)}{(4ct_r)^2}\left(\alpha_a(2K-k) + k(f+F)\right)}^{\text{reduced cost of quality provision}} \tag{OA.12}$$

When α_a is arbitrarily large, the increment in welfare is positive. This proves the first part of the proposition. It results from (OA.12) that:

$$\frac{1}{k}\frac{\partial W(k,f,F)}{\partial f} = \frac{K+k}{4ct_r} - \frac{\alpha_a(K-k) + k(f+F)}{4ct_r^2}, \tag{OA.13}$$

Welfare-maximization relative to F results in the same relation. First-order conditions are $f + F = 2f_0(k)$ with $f_0(k) \equiv \frac{1}{2k}\left((K+k)t_r - \alpha_a(K-k)\right)$ and $\frac{\partial^2 W(k,f,F)}{\partial f^2} < 0$. This means $f + F = 2f_0(k)$ are candidate optimal fees. We need to ensure these fees belong to $\mathscr{F}$:

- if $f_0(k) < \underline{f}$, as is always the case when there is little differentiation ($t_r < \alpha_a$) and k is small, a social planner aims at relaxing competition and sets the smallest feasible fees $f = F = \underline{f}$.
- if $\underline{f} \leq f_0(k) \leq \bar{F}$, any feasible fees such that $f + F = 2f_0(k)$ maximizes welfare. This set is nonempty because $f = F = f_0(k)$ is feasible and meets the condition.

- if $f_0(k) > \bar{F}$, as is always the case when there is strong differentiation ($t_r > \alpha_a$) and k is small, $f = F = \bar{F}$ maximizes welfare.

OA.1.6 The case of bundling

When go-betweens bundle their services, i.e., platforms have to accept to both sponsor and publish, or reject any deal, the feasible set is not bounded from below by platform's participation constraint. Instead, A1 may become become binding.

Example 1 (Bundling of sponsorship and publishing) *Assume A1. If the go-between bundles its publication and sponsoring services, any fees (f, F) such that $f = F \leq \frac{\alpha_a}{2}$ are feasible.*

Proof. The proof is easily derived from the analysis of platforms equilibrium profits (8), and the observation that references actually occur if and only if both platforms accept the sponsoring bundle. In that case, it is a dominant strategy to accept the deal as soon as expected future equilibrium profits are greater with than without sponsoring. If k is small, we can express the maximum sum of fees $\bar{F}$ such that go-betweens make a nonnegative margin $m \equiv F - f$. Any fees such that $f + F < \bar{F} = \alpha_a + \frac{4ct_r^2}{K\alpha_a}(\alpha_a - m)$ belong to the feasible set, as long as they are not as high as to violate assumption A1. ■

OA.1.7 Exclusive contracts and participation fees

Over the course of 2018, some major press groups (Le Monde, Le Figaro, 20minutes and others) have signed exclusive deals with specific go-betweens. These deals entail not only content recommendation to external sources but also self-promotion. They are inherently different from the base service studied in the main body of the article. In particular, it is reported that these partnerships entail payment guarantees. Ancillary services such as free analytics of reader traffic can also be seen as a form of lump-sum payment from the go-between to platforms. Hence we now consider the case when sponsoring services are exclusive, and we allow for the go-between to set not only per-click fees f and F (commonly referred to as "usage fees" in the literature on two-sided markets), but also some participation fees p_p and p_s. p_p is the lump-sum payment charged to a publisher who displays sponsored links. p_s is the lump-sum payment charged to potential sponsors. We allow these lump-sum payments to be negative to account for, e.g., the free analytics services that the go-between may provide.

To enter, E must subsidize one group, say the group of sponsors such that they accept to join the platform. For this to happen, he needs to set the entry fee sufficiently low such that they are willing to join, even if it results in them not being matched at all:

$$\overbrace{-p_s^E}^{\text{subsidy to join new entrant}} \leq \overbrace{(\bar{F} - F^I)k - p_s^I}^{\text{surplus made with incumbent}} \tag{OA.14}$$

In the case of strict inequality the entrant attracts all sponsors, $z_s^E = 1$. Platform E will subsequently benefit from the intergroup externality when it courts publishers. The latter rationally expects all sponsors to be enrolled with the entrant, who needs to provide a payment

$$(f^E - \underline{f})k - p_p^E \geq -p_p^I, \tag{OA.15}$$

so that publishers join the platform. The entrant can set $F^E = \bar{F}$, as it does not affect the participation of either side – see equations (OA.14) and (OA.15). To prevent entry, the incumbent must choose its pricing such that the profits of the entrant are nonpositive:

$$\Pi^E(f^E, F^E, p_p^E, p_s^E) = p_s^E + p_p^E + (\bar{F} - f^E)k \leq 0 \tag{OA.16}$$

Profit maximization of the entrant results in conditions (OA.14) and (OA.15) being met with equality. Inserting them into (OA.16) yields condition:

$$\Pi^E(f^E, F^E, p_p^E, p_s^E) = \Pi^I(f^I, F^I, p_p^I, p_s^I) + (f^I - \underline{f})k \leq 0 \tag{OA.17}$$

For this condition to allow the incumbent to make strictly positive profits, one would need the fee charged to hosts by the incumbent f^I to be less than $\underline{f}$. However, in that case no platform would be willing to publish external links. Hence (OA.17) cannot be negative without the incument's profits being negative. Similarly to the case with no participation fees, this means that preventing entry deterrence is possible, and requires the incumbent to make no profits. The incumbent optimally sets $f^I = \underline{f}$. Thus, we have that there exist only equilibria that provide efficient matching (all sponsors and publishers are with the same go-between), with a unique active firm making zero profit ($p_p + p_s + (F - f)k = 0$). However, the resulting fees are $f = \underline{f}$ and F may lie anywhere in $[\underline{f}, \bar{F}]$, which does not induce the maximum quality in the long run. The multiplicity of equilibria can be reduced by enforcing that payments be per-interaction only, or allowing for heterogeneity in the trading behavior of agents (see Reisinger (2014)).

OA.2 Type-A users single-home

Structural reasons such as regulation, exclusivity contracts, or limited financial resources may constrain A-users to enroll with at most one platform.

OA.2.1 Competition in the market: impact of interplatform references on quality and welfare

OA.2.1.1 Stage 3 and 2

A-users: A natural setting when single-homing is likely to arise is when advertisement is informative. In that case advertisers are interested in interacting once with each user, but attach no value to further impressions on the same user. Denote $h(K) \equiv 1-(1-\rho)^{\frac{K}{\rho}}$ and $h(k) = 1-(1-\rho)^{\frac{k}{\rho}}$. $h(K)$ ($h(k)$) is the probability that a given user views at least one item in its anchor platform (platform of second choice). The utility of an A-user joining platform i is:

$$U_a^i(x,k) = \alpha_a \left(n_r^i h(K) + n_r^j h(k)\right) - t_a \mid x - x_i \mid - p_i \tag{OA.18}$$

α_a is the intrinsic benefit that A-user enjoys from informing an R-user. The A-user utility is proportional to the number of R-users who are on the platform, and the probability that he interacts with each of them at least once. The first term in (OA.18) corresponds to the number of interactions with R-users anchored in i ($n_r^i h(K)$) and those anchored in j but who may roam to i ($n_r^j h(k)$). Preference cost $t_a x$ can have various justifications. It may for example represent how much a brand wants to be associated with the image of such or such platform. In the media application, it may also represent the preference of advertisers over some platform-specific attributes: newspapers may offer advertisement slots in the form of videos, or in the form of inserts or personalized page backgrounds. Each of these formats may be especially fit for certain types marketing campaigns. p_i is a participation fee paid by the A-user to the platform.

R-users: These users are individuals searching for information or a service. Take a user is located in x. Her utility when she anchors with platform i and roams to platform j to access additional content is similar to (1):

$$\begin{aligned} U_r^i(x,k) =& \bar{u}_r - (1+MH)s + (q_i - t_r \mid x - x_i \mid)K + (q_j - t_r \mid x - x_j \mid) k \\ &+ \alpha_r n_a^i h(K) + \alpha_r n_a^j h(k) \end{aligned} \tag{OA.19}$$

Compared to (1), an extra term $\alpha_r n_a^i h(K)$ is added. It corresponds to the benefit a reader derives from being informed of the advertisement. α_r can also be interpreted as the benefit of seeing informative advertisement, instead of the persuasive advertisement that may have been displayed instead.

We assume again that users always get nonnegative surplus from reading any article, even from the platform least close to their preferences. We will see further that this requires the cost of provision of quality be not too high. For simplicity we assume again that in the benchmark case when references are absent, type-R users single-home.

Platforms: Platforms' profits are similar to the multi-homing case, expect that advertisement is not charged for each interaction, but for participation:

$$\Pi_i(p_i, q_i, p_j, q_j) = p_i n_a^i + f k n_r^i - F k n_r^j - c_i q_i^2 \tag{OA.20}$$

For formal simplicity and ease of comparison with the body of the paper, we assume that ρ is small and therefore $h(K) = K$ and $h(k) = k$. Further, we define the following assumptions:

A2: $t_r t_a \geq \alpha_r \alpha_a (h(K) - h(k))$

A3: $t_r^2 t_a \geq \alpha_a (h(K) - h(k)) \left(\alpha_a \frac{K-k}{8c} + t_r \alpha_r\right)$

A4: $c < \frac{\alpha_a h(K) - h(k)(\alpha_a - f - F)}{4t_r^2}$

Assumptions A2 and A3 ensure that platforms are sufficiently differentiated (t_r and t_a large enough) for a market-sharing equilibrium to exist. Note that A3 implies A2. A4 is similar to A1. It states that the costs of provision of quality must be sufficiently low so that equilibrium quality induces all R-users to roam whenever there are references. With the utility functions defined in (OA.18) and (OA.19), we can solve stage 3 of the game. The number of users anchored in i is :

$$n_r^i = \frac{1}{2} + \frac{q_i - q_j + \alpha_r(n_a^i - n_a^j)}{2t_r} \tag{OA.21}$$

$$n_a^i = \frac{1}{2} + \frac{p_j - p_i + \alpha_a(h(K) - h(k))(n_r^i - n_r^j)}{2t_a} \tag{OA.22}$$

With this observation, we turn to stage 2. Proposition 2 describes the unique equilibrium in stage 2 of the game when fees are feasible.

Proposition 1 *Assume A2 to A4 hold and $(f, F) \in \mathscr{F}$. There exist a unique Nash equilibrium. It is symmetric. Quality and prices are given by the following relations :*

$$q_i(k) = q(0) - h(k)\frac{\alpha_a - (f + F)}{4ct_r} \tag{OA.23}$$

$$p_i(k) = p(0) + h(k)\alpha_r\frac{\alpha_a - (f + F)}{t_r} \tag{OA.24}$$

, where $q(0) = \frac{\alpha_a K}{4ct_r}$ and $p(0) = t_a - \frac{\alpha_r \alpha_a K}{t_r}$ are respectively the equilibrium quality and prices, when there are no interplatform references ($k = 0$).

Proof. Relation (OA.21) combined with (OA.22) yields that the number of users of type-R and A on platform i is:

$$n_r^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_r(p_j - p_i) + t_a(q_i - q_j)}{t_r t_a - \alpha_r \alpha_a(h(K) - h(k))} \tag{OA.25}$$

$$n_a^i = \frac{1}{2} + \frac{1}{2}\frac{\alpha_a(h(K) - h(k))(q_i - q_j) + t_r(p_j - p_i)}{t_r t_a - \alpha_r \alpha_a(h(K) - h(k))} \tag{OA.26}$$

Profit maximization, assuming both platforms have the same costs of production

$(c_i = c_j = c)$, and using the symmetry of the model yields:

$$2cq_i = p_i \frac{\alpha_a(h(K) - h(k))}{2(t_r t_a - \alpha_r \alpha_a(h(K) - h(k)))} + k(f + F)\frac{t_a}{2(t_r t_a - \alpha_r \alpha_a(h(K) - h(k)))} \tag{F.O.C. q_i}$$

$$p_i \frac{t_r}{2(t_r t_a - \alpha_r \alpha_a(h(K) - h(k)))} = n_a^i - k(f + F)\frac{\alpha_r}{2(t_r t_a - \alpha_r \alpha_a(h(K) - h(k)))} \tag{F.O.C. p_i}$$

, which simplifies into relations (9) and (10). For this first condition approach to be valid, one needs to check that $\frac{\partial^2 \Pi_i}{\partial q_i^2} < 0$ and that the determinant of the Hessian is positive. Tedious calculations show that these hold under assumptions A2 and A3. ■

We observe again that if f and F are small, competition for both types of users is softened: quality, which is prized by R-users, decreases with the number of sponsored links k. A similar interpretation holds for the price p_i paid by A-users that increase with k unless $f + F$ is sufficiently high.

OA.2.1.2 Stage 1: Setting of fees

In stage 2, platforms take fees as given. The setting of fees in stage 1 depends on who (users, platforms, independent third parties or a regulator) controls the referencing activity of the go-between. We first analyze user welfare, the profits of platforms, and social surplus as a function of these fees.

Surplus analysis: We first calculate the incremental utility $\Delta U_l^i(x, k) \equiv U_l^i(x, k) - U_l^i(x, 0)$ received by a user of type l, located at x and having joined platform i.

R-user surplus: We calculate the equilibrium R-user surplus, normalized by the number of references:

$$\frac{1}{k}\Delta U_r^i(x,k) = K\overbrace{(q_i(k)-q_i(0))}^{\text{quality effect}}+k\overbrace{\left(q_i(k)+\frac{\alpha_r}{2}-t(1-x)\right)}^{\text{diversification effect}}$$

using that $q^*(0)=\frac{\alpha_a K}{4ct_r}$:

$$\frac{1}{k}\Delta U_r^i(x,k) = -\frac{k}{K}q^*+(K+k)\frac{f+F}{4ct_r}+\frac{\alpha_r}{2}-t_r(1-x) \tag{OA.27}$$

A first immediate result is that despite the increase in content diversity, the presence of references results in a decrease in utility for all R-users whenever fees $f+F$ are below a threshold $F^*\equiv\frac{k\alpha_a+2(t_r-\alpha_r)ct_r}{K+k}$. Surprisingly, this is true for all users, including those with weak preferences ($x=\frac{1}{2}$), who are those who enjoy the most the content of their platform of second choice. This is due to the fact the decrease in quality does not only affect the k original content, but the whole corpus of content. Figure 5 illustrates the effect of f and F on user surplus.

Previous discussions and the analysis of relation (OA.27) show Proposition 4 carries over to the case when A-users are single-homer interested in unique interactions.

A-user surplus: A-users receive incremental utility through more matches with users of the other side. However, inter-plaform references also induces a price increase. Overall, the incremental utility is:

$$\frac{1}{k}\Delta U_r^i(x,k) = \overbrace{\frac{\alpha_a}{2}}^{\text{more matches}} - \alpha_r\overbrace{\frac{\alpha_a-(f+F)}{t_r}}^{\text{price effect}}$$

Again, we observe that $f=F=\bar{F}$ is the set of feasible fees that maximize A-user surplus. If fees are set at marginal costs 0 and $\alpha_r>\frac{t_r}{2}$, the surplus of A-users decreases despite more interactions with R-users.

Platform profits: The following proposition mirrors Proposition 5.

Proposition 2 *Assume that reference fees* (f, F) *belong to the feasible set* $\mathscr{F}$*. Platform profits always increase with interplatform references. Ex ante bilateral negotiation between platforms results in fees that maximize the platform surplus:* $f = F = \underline{f}$*.*

Proof. Denoting $\Delta\Pi_i(k) \equiv \Pi_i(k) - \Pi(0)$ the increment in equilibrium profits of each symmetric platform when there are references:

$$\Pi_i(k) = \Pi_i(0) + \frac{1}{2}(p_i - p^* + f - F) + c\left((q(0))^2 - q^2(k)\right)$$

this implies that incremental profit, normalized by the number of references is:

$$\frac{1}{k}\Delta\Pi_i(k) = \frac{\alpha_r}{2t_r}(\alpha_a - (f+F)) - \frac{F-f}{2} + c\frac{\alpha_a - (f+F)}{(4ct_r)^2}(\alpha_a(2K-k) + k(f+F)) \tag{OA.28}$$

Following the same steps as in Appendix OA.1.4, it is easy to show that $F = f = \underline{f}$ is the set of feasible fees that maximizes the joint profits of platforms. Additionally, $\frac{\partial \Pi_i}{\partial k}(k) > 0$ for all feasible fees, meaning references are always profitable. ■

Note that the fees selected by platforms in case of bilateral negotiation are those that minimize quality and user surplus.

Social surplus : We assume that the social planner is unbiased. Denote $\Delta W(k) \equiv W(k) - W(0)$ the increment in social welfare due to references. We derive:

$$\frac{\Delta W(k)}{k} = \overbrace{\frac{\alpha_a + \alpha_r}{2}}^{\text{additional network effects}} - \overbrace{\frac{3t_r}{4}}^{\text{average distance to second-choice platform}} - \overbrace{\frac{k}{K}q^*}^{\text{quality variation}} \tag{OA.29}$$

$$+ \overbrace{(f+F)\frac{K+k}{4ct_r}}^{\text{procompetitive fees}} + \overbrace{2c\frac{\alpha_a - (f+F)}{(4ct_r)^2}\left(\alpha_a(2K-k) + k(f+F)\right)}^{\text{reduced cost of quality provision}} \tag{OA.30}$$

We derive that:

$$\frac{\partial W(k)}{\partial f} = k\frac{K+k}{4ct_r} - \frac{ck}{4(ct_r)^2}\left(\alpha_a(h(K) - h(k)) + k(f+F)\right) \tag{OA.31}$$

, and $\frac{\partial^2 W(k)}{\partial f^2} < 0$. First-order conditions yield that $f + F = 2f_0(k)$ with $f_0(k) \equiv \frac{1}{2k}\left((K+k)t_r - \alpha_a(h(K) - h(k))\right)$, the set of fees that maximize surplus, assuming these fees belong to $\mathscr{F}$. Welfare maximization relative to F results in the same relation. If $\underline{f} \leq f_0(k) \leq \bar{F}$, any feasible fees such that $f + F = 2f_0(k)$ maximizes welfare. This set is nonempty since $f = F = f_0(k)/2$ is feasible and meets the condition. If $t_r < \alpha_a$, meaning platforms are not very differentiated, and k is small, a social planner would aim at relaxing competition and set the smallest feasible fees $f = F = \underline{f}$ when platforms are not very differentiated on side R. Conversely if $f_0(k) > \bar{F}$, as is always the case when there is strong differentiation ($t_r > \alpha_a$), $f = F = \bar{F}$ maximizes welfare.

OA.2.2 Competition for the market: impact of interplatform references on entry

The previous section analyzed the effects of references on competition between established platforms. However, because they exploit network effects, it is a well-established fact that platform markets tend to be very concentrated (see for example Evans and

Schmalensee (2016)): many markets are dominated by a few players who, once established, may exploit their market power and become particularly hard to contest. This is especially likely to happen when platforms are poorly differentiated. In that case, the relevant paradigm is competition for the market. We question whether sponsored content may allow a more efficient platform to win the market.

Interplatform references may seem to favor entry, as users are encouraged to view the content of potential entrants. In fact, they may even inform users of the existence of a competitor. In this section, we show instead that cross-referencing between an incumbent and an entrant may further impede entry.

Assuming that the entrant is ex ante excluded from the already existing network of a go-between would straightforwardly result in concluding that references impede entry: utility function (OA.19) is monotonically increasing in k. Hence, references constitute a natural barrier to entry of platforms excluded from a go-between's network of publishers and sponsors.[2] We rule out this obvious case to concentrate on the more subtle one when both the entrant and an incumbent belong to the same network of sponsors/publishers and references are reciprocal. This well represents the business model of content discovery platforms who manage content suggestions on behalf of many online news outlets.

We assume that platform 1 originally benefits from favorable expectations: users believe the other side has enrolled with platform 1. Platform 1 is therefore formally equivalent to an incumbent whose users make participation decision based on present market shares. We question under which conditions platform 2, who endures unfavorable beliefs but is more efficient ($c_2 \leq c_1$) may attract positive market shares.

The timing of the game is same as in Section 3. Stage 3 is however decomposed into two sub-stages, to take account explicitly of the dynamics of users' migration between

[2]It often occurs that platforms 1 and 2 are both subsidiaries of the same company. Figure 12 in Appendix A shows that search engine Kayak refers to sister company Rentalcar. Both are owned by parent company Booking Holdings. In that case, keeping quality constant, interplatform references undoubtedly increase user utility thanks to an increase in the quantity of content accessible to users. This also means that these links constitute a strong barrier to entry if incumbents do not refer the content of potential entrants.

platforms.

1. Go-between fixes reference fees. Platforms accept or reject to publish/sponsor.

2. Platforms simultaneously choose quality q_i and price p_i.

3. Users observe q_i, p_i, choose their "anchor" platform. R-users may then "roam" to the rival. This stage is decomposed into two periods, which differ by the initial belief of users:

   3d. Users have initial beliefs, favorable to platform 1. They may decide to migrate.

   3c. Users observe the (potentially) new market shares. They may decide to migrate.

Period $3d$ corresponds to the "Divide" part and lasts δ, while $3c$ consists in the "Conquest" of the other side and lasts $1 - \delta$. A positive δ captures the fact that migration to a platform, and observability of such migration takes time. It represents how much inertia is involved in the migration process. During period $3d$, each side may join a different platform, which means references will indeed be used and reference fees may be charged to platforms. Throughout this section, we will assume that δ is "small", to focus on the marginal effects of the migration inertia while keeping the model simple. $\delta = 0$ corresponds to the timing of Section 3. We again look for subgame-perfect Nash equilibria. For these to exist we need to assume that quality is actually provided for by a platform if and only it attracts at least one user.

OA.2.2.1 Entry of a differentiated platform

Condition for entry of a differentiated platform can be easily derived from the results of Section 4. Once platform 2 has entered, he receives profits $\Pi_i(k)$ (see equation (8), and Appendix B in the asymmetric cost case). For entry to occur, one simply needs to verify that $\Pi_2(k) > 0$. If costs are symmetric we can show that for 2 to enter and secure positive market shares, costs must be greater than c_{lim} defined as:

$$c_{lim} = \left(8t_r\left(\frac{t_a t_r - (F-f)\alpha_r}{\alpha_a(h(K)-h(k)) + (f+F)k} - \alpha_r\right)\right)^{-1}$$

We observe that c_{lim} decreases with k. The smaller the fee margin $m \equiv F - f$ and the smaller the sum of fees $f + F$, the smaller c_{lim}. The intuition for this is rather straightforward: references increase the time spent by users on either platforms –and their exposure to A-users. There is therefore a creation of value which encourages entry. As we observed in the previous section, low reference fees increase profits of each platform and therefore encourage entry. The fee margin m corresponds to value extracted by the go-between and as such decreases the attractiveness of entry. We conclude that references encourage entry of a differentiated platform, especially if fees are low. This conforms to the intuition, since references induce an expansion of the total number of interactions and, in turn, the profitability of the market. We will now observe that this conclusion may be reversed when platforms are undifferentiated and compete for the whole market.

OA.2.2.2 Entry of a non-differentiated platform

The existence of market-sharing equilibria rests on the assumption that both platforms make nonnegative profits in equilibrium, which is guaranteed by assumption A5:

A5: $t_r^2 \geq \frac{\alpha_a}{8c}\frac{(h(K)-h(k))^2}{K+k}$

However, if platforms are insufficiently differentiated, equilibrium may require that only one platform be active. For the sake of simplicity, we consider the case when market participants are perfectly homogeneous ($t_r = 0$), thereby violating assumptions A3. Platform 2 is therefore a "sibling" of 1 in the sense he caters to the very same readership as 1. Their only differences are potentially different costs of quality provision, and the fact k articles are original to each platform. Similarly to the previous section, we assume that reference fees f and F are decided ex ante, and thus platforms take them as given. To win the market, 2 has three options. The most direct is to offer both an attractive prices to A-users and high quality to R-users. This strategy, which we denote by"AR" is fast and allows 2 to become focal rapidly, since both sides migrate as soon as in period $3d$. This is, however, a costly approach since network effects have to be overcome on both sides of the market. 2 could instead employ a Divide-and-Conquer (DC) strategy (Caillaud and Jullien, 2003): 2 may target first A-users (strategy "A"), or type-R (strategy "R") by offering them a very good deal so they migrate in period $3d$, and then cater to the other side of the market.

Following Doganoglu (2003), Mitchell and Skrzypacz (2006), Markovich (2008), we assume users are naive and their decisions are based on observed market shares at the moment they decide which platform to join. This gives 1 an advantage in the sense 2 will have to engage in very fierce price or quality competition in order to convince a first side of users to switch in stage $3d$.

For simplicity we assume again that costs c_1 and c_2 are not too high so quality in equilibrium always exceeds network effects α_r. We assume A4 holds:

A4: $c_1, c_2 \leq \frac{2\alpha_a(h(K)-h(k))}{\alpha_r^2}$

We solve again the game by backward induction, starting from stage 3.

stage 3: decision of users The instantaneous utility of users is same as in Section 3, with the simplifying assumption that users are homogeneous ($t_r = t_a = 0$):

$$U_r^i = \bar{u}_r + (q_i + \alpha_r n_a^i)K + (q_j + \alpha_r n_a^j)k \tag{OA.32}$$

R-users choose 2 if and only if $U_r^2 \geq U_r^1$, or $q_2 - q_1 \geq \alpha_r(2n_a^1 - 1)$. At the beginning of period $3d$, we have that $n_a^1 = 1$. Hence users switch to platform 2 in stage $3d$ if and only if the quality of 2 largely exceeds that of 1:

$$q_2 \geq q_1 + \alpha_r \equiv \overline{q}_2 \tag{OA.33}$$

, meaning network effects award 1 an advantage. If A-users have switched in period $3d$, R-user switch in period $3c$ if and only if :

$$q_2 \geq q_1 - \alpha_r \equiv \underline{q}_2 \tag{OA.34}$$

Let us now turn to A-users. Their utility if they join i instead of j is:

$$U_a^i = \bar{u}_a + \alpha_a n_r^i K + \alpha_a n_r^j k - p_i \tag{OA.35}$$

Advertisers switch to platform 2 if and only if $p_2 - p_1 \leq -\alpha_a(K-k)(2n_a^1 - 1)$. Attracting A-users in period $3d$ requires 2 to post a price lower than p_1:

$$p_2 \leq p_1 - \alpha_a(K-k) \equiv \underline{p}_2 \tag{OA.36}$$

If R-users are already with platform 2 in period $3d$, A-users migrate if and only if:

$$p_2 \leq p_1 + \alpha_a(K-k) \equiv \overline{p}_2 \tag{OA.37}$$

These expressions assume $q_1 \geq \alpha_r$, which we will see is guaranteed by assumption A4. Again, we observe that 1 is advantaged by network effects at the "divide" stage $3d$ (see penetration strategies (OA.33) and (OA.36)). However, this advantage vanishes as content on the two platforms are less redundant, and there are more references (k increases). The switching conditions for pioneer movers (OA.33) and (OA.36) show an important difference: only type-A's switching decision is impacted by interplatform references k: references help courting A-users, but are ineffective in attracting R-users.

This means references facilitate entry through the A strategy (courting A-users first) as $\underline{p}_2$ increases with k. However, R strategies become more difficult since the "Divide" is unchanged while "Conquer" is more costly ($\overline{p}_2$ decreases with k).

Stage 2: best response of 2 We now turn to Stage 2, when platforms 1 and 2 need to set their quality and price. Taking the menu of 1 as given, 2 can engage either in strategy AR, A, R or not enter (NE). Due to the fact users are homogeneous, we saw above that only a few price and quality strategies (p_2, q_2) are not strictly dominated by other strategies. Candidate prices in a strategy that involves entry of 2 are $\underline{p}_2$ and $\overline{p}_2$. Candidate qualities are $\underline{q}_2$ and $\overline{q}_2$. 2's outside option is not to enter, which he achieves by setting $(q_2 = 0, p_2 = +\infty)$.

AR strategy: The AR strategy consists in choosing both a high quality and a low price, to ensure that all users enrol as soon as in the first subperiod $3d$. This results in profits:

$$\pi_2(AR, q_1, p_1) = \left(\underline{p}_2 - c_2(\overline{q}_2)^2\right) + \delta f k \tag{OA.38}$$

These are type-A participation fees minus costs of quality provision. The second term corresponds to revenues harvested through referencing in period $3d$: all R-users being with 2 during this period, there is interplatform referencing and the platform that hosts R-users is rewarded f per click to external content.

R strategy: 2 may cater first to R-users in period $3d$ and wait for A-users to observe the new market shares in period $3c$. In that case, he needs to set a high quality $\overline{q}_2$ and may maintain a relatively high price to A-users $\overline{p}_2$. Profits are:

$$\pi_2(R, q_1, p_1) = (1 - \delta)\overline{p}_2 - c_2(\overline{q}_2)^2 + \delta f k \tag{OA.39}$$

The first term is profit made with A-user subscriptions. These are collected in the "conquest" period $3c$ only. The second term is the cost of high quality provision to attract R-users in period $3d$. R-users may use references and roam to platform 1 in period $3c$ which yields additional benefits δfk to 2.

A strategy: if platform 2 caters first to A-users in period $3d$, he sets a low price $\underline{p}_2$ and may maintain a relatively low quality $\underline{q}_2$. Profits are:

$$\pi_2(A, q_1, p_1) = \left(\underline{p}_2 - c_2(\underline{q}_2)^2\right) - F\delta k \tag{OA.40}$$

The second term corresponds to fees the entrant has to pay in order to have R-users (who are still with 1 in period $3d$) visit its page and interact with its own A-users. It is immediate that choosing both a high price and a low quality results in no switching and no referencing, and therefore this strategy is weakly dominated by the NE strategy where no quality is provided at all by the entrant. For entry to occur, we need at least one of profits (OA.38), (OA.39) and (OA.40) to be greater than 0, which is the value of 2's outside option NE. An analysis of the dependency of profits functions to k yields the following proposition:

Proposition 3 *Fix the strategy of the platform that benefits from favorable beliefs* (q_1, p_1).

- *Interplatform references renders a direct entry through AR strategy easier. Similarly, a DC strategy starting with the side whose switching decisions is most positively affected by references (side A) is facilitated.*
- *Entry by a DC strategy starting by the least sensitive side R is made more difficult.*
- *High sponsor fees F render an A strategy more difficult while high publisher fee f leads to more entry by both AR and R strategies.*

Proof. Follows from the analysis of the profit functions (OA.38) to (OA.40). ■

We can now define which of these 4 decisions (NE, AR, R or A) is 2's best response to platform 1's posted prices and quality. A comparison of (OA.38) and (OA.39) shows that for δ small enough, the AR strategy is dominated by R. In the remainder of the paper, we therefore focus on A and R strategies. 2 prefers R over A if and only if

$$4c_2q_1\alpha_r + \delta p_1 \leq \alpha_a(h(K) - h(k))(2 - \delta) - \delta(F - f)k \tag{OA.41}$$

This means targeting R-users makes sense if and only if q_1 is sufficiently low (meaning R-users will be relatively easy to attract) and p_1 is low (making the A strategy too costly). Otherwise, A is preferred over R. Note also that only prices below the willingness to pay of A-users are relevant to our analysis: $p_1 \leq \bar{u}_a + \alpha_a K \equiv \bar{p}$. Figure 2 provides a graphical illustration of these results. As we will see in the next section, the black dot is a candidate for 1's best entry deterrence strategy, while the red and blue dots represent candidate accommodation strategies.

[Figure 2 about here.]

Stage 2: best response of 1 We now characterize the best response of platform 1 to any profitable menu posted by 2. We then show that the menu of 1 together with 2's most constraining reply constitute a Nash equilibrium of stage 2. Importantly, when 1 cannot deter 2 to enter and capture the market, 1 may nevertheless "steer" the entrant towards a given entry strategy. For example, when continuation payoffs are such that it is unprofitable for 1 to deter entry, 1 may still decide to post a relatively low price such that 2 enters through R instead of A. That way, 1 can retain the valuable A-users in period $3d$.

Profits of 1 when entry is accommodated. To determine platform 1's optimal strategies, we first investigate its profits when he accommodates entry. If (OA.41) is not verified, 1 accommodates entry while steering 2 into adopting an A strategy. 2 enters by attracting A-users first and platform 1 receives some payment in period $3d$ from interplatform referencing. 1's profits are:

$$\pi_1(A, p_1, q_1) = \delta f k - c_1(q_1) \tag{OA.42}$$

Profit is maximized when q_1 is minimized, i.e., is such that (OA.41) is met with equality with $p_1 = \bar{p}$:

$$\pi_1^*(A) = \delta f k - c_1 \left(\frac{2\alpha_a(h(K) - h(k)) + \delta(\bar{p} - (f + F))}{4c_2\alpha_r} \right)^2 , \tag{OA.43}$$

where the star superscript denotes the maximum profits of 1, given that entry by A-users is accommodated.

If (OA.41) is not verified, 1 accommodates entry while steering 2 into adopting an R strategy. Platform 2 enters and platform 1 still receives some payment from its own A-users in period $3d$ but also needs to pay for interplatform referencing, so its A-users do have access to some R-users. 1's profits are:

$$\pi_1(R, p_1, q_1) = \delta(p_1 - Fk) - c_1(q_1)^2 \tag{OA.44}$$

It follows that platform 1, conditionally on choosing to accommodate entry through the R side, sets $q_1 = 0$ and $p_1 = min(\bar{p}, p_0(k))$, with $p_0(k)$ the price that verifies (OA.41) with equality when $q_1 = 0$:

$$\pi_1(R, p_1, q_1) = \delta(min(p_0(k), \bar{p}) - Fk) \tag{OA.45}$$

It follows from (OA.43) and (OA.45) that if k is small enough, accommodation by A is dominated by accommodation by R. Hence we will focus on R as the only relevant accommodation strategy.

Profits of 1 when entry is deterred. We now turn to the case when 1 deters entry. This case is more complex, owing to the fact platform 1 needs to ensure both entry strategies R and A are unprofitable for platform 2. Proposition 4 describes platform 1's best deterrence strategy.

Proposition 4 *Assume A4 holds. The maximum profit of an incumbent deterring entry is:*

$$\pi_1^*(NE) = \alpha_a(h(K) - h(k)) + c_2(\tilde{q}_1 - \alpha_r)^2 - c_1(\tilde{q}_1)^2 + Fk\delta\,, \tag{OA.46}$$

where quality $\tilde{q}_1$ *is implicitly defined by :*

$$2\alpha_a(h(K) - h(k)) + \delta k\left(\frac{f}{1-\delta} + F\right) + c_2\left((\tilde{q}_1 - \alpha_r)^2 - \frac{(\tilde{q}_1 + \alpha_r)^2}{1-\delta}\right) = 0 \tag{OA.47}$$

Proof. We propose here a sketch of the proof. A rigorous proof using the method of Lagrange multipliers is proposed in Appendix OA.2.4.1.

We saw in stage 2 that when both p_1 and q_1 are high the best entry strategy is A. Conditional on 2's best reply being to enter with strategy A we can show that 1 should aim at minimizing q_1. Conversely when both p_1 and q_1 are small and the binding entry constraint is the R strategy, 1 aims at maximizing q_1. Hence, the optimal choice for 1 is to set q_1 and p_1 such that both entry conditions R and A are met with equality, thereby ensuring that there is no entry, while maximizing profits. We can show that if the binding constraint is 2's threat of entry by R, 1 increases quality and price until A becomes binding. We then show that if the binding constraint is 2's threat of entry by A, 1 decreases quality and price until R becomes binding. We conclude that the optimum quality and price is such that both entry conditions are binding : (OA.39) = (OA.40) = 0. From these equalities follow that platform 1 sets quality $\tilde{q}_1$ implicitly defined by

(OA.47) and price is

$$\tilde{p}_1 = \alpha_a(h(K) - h(k)) + \delta F k + c_2(\tilde{q}_1 - \alpha_r)^2 \tag{OA.48}$$

From this, we derive profits (OA.46). ■

From Proposition 4 we observe that if migration is instantaneous ($\delta = 0$) and 1 and 2 are symmetric, 1 is the unique platform that serves the whole market. It makes strictly positive profits $c_1\alpha_r^2$. From this proposition and the observation that there will be entry if and only if $\pi_1^*(NE) < max(0, \pi_1^*(R), \pi_1^*(A))$, we derive the following Corollaries 1 and 2.

Corollary 1 *Fix reference fees (f, F). As references become more and more frequent (k increases) entry by A strategy is facilitated while entry by R is further impeded. When the platform with unfavorable beliefs has a cost advantage, the second effect prevails.*

Proof. See Appendix OA.2.4.2 ■ Corollary 1 shows that, perhaps surprisingly and due to network effects, interplatform references between an incumbent and an entrant can impede entry. We now show that the profits of platforms decrease as the fee margin $F - f$ increase.

Corollary 2 *Fix the number of references k. A large publisher fee f and a small sponsor fee F favor entry. Fixing the Go-between margin to $m = F - f$, entry is facilitated by high fees if and only if the externality from A to R is large enough.*

Proof. See Appendix OA.2.4.3 ■

A direct implication of Corollary 2 is that promoting competition among go-getween, which decreases the fee margin, promotes entry. Hence we showed that when an incumbent has a cost disadvantage, interplatform referencing deters entry.

OA.2.2.3 Competition for the market: policy implications

To sum up, this section notes that in two-sided markets, network effects generally favor the incumbent and render entry difficult. However, intermediation between platforms in the form of references modify entry incentives.

When an entrant is sufficiently differentiated from the incumbent, references favor entry, especially if reference fees are low. When a more efficient –but undifferentiated– platform considers entry, references generally favor entry by simultaneous attraction of both sides of the market (AR strategy). When simultaneous entry is not feasible, a DC approach may be undertaken. References between platforms make the division easier, but also make the conquest more difficult. In particular, it always facilitates entry by a DC strategy starting from the side whose choice is strongly affected by references. However, it may render the other DC strategy (targeting first the side whose choice is less affected by references) more difficult.

In our case the second effect prevails. The policy implication is that if a regulator observes that strong network effects impede the entry of a competitor despite referencing he may want to investigate the nature of network effects. This is, however, a two-edged sword: if references don't impact a side's choice of a platform, references may make some DC strategies ineffective and reinforce the entry barrier, as is the case in our model.

In all situations, a large positive margin $F - f$ between the fees renders entry more difficult. Hence a social planner willing to encourage entry should strive to promote competition among go-betweeens and decrease the fee margins. High fee margins are desirable only if entry is deemed excessive.

OA.2.3 Transaction fees

The main text assumes platforms only offered fixed participation fees p_i, that do not depend explicitly on how well the platform is doing on type-R side. In practice however, the pricing scheme could be a function of how many interactions happened the platform.

For example in the media industry, a fee can be charged per-impression or per-click. The present section analyzes equilibria when platforms can charge both a fixed component p_i for participation and a per-transaction fee γ_i.

The utility of R-users and their demand function are the same as in (1) and (8), respectively. Utility of type-A now includes interaction fees:

$$U_a^1 = \bar{u}_a + (\alpha_a - \gamma_i)\left(n_r^1 K + n_r^2 k\right) - t_a x - p_i \tag{OA.49}$$

From this, we establish the demand function of A-users:

$$n_a^i = \frac{1}{2} + \frac{p_j - p_i + (\alpha_a - \gamma_i)\left(n_r^i K + (1 - n_r^i)k\right) - (\alpha_a - \gamma_j)\left((1 - n_r^i)K + n_r^i k\right)}{2t_a} \tag{OA.50}$$

Solving the system of equations (8) and (OA.50):

$$n_a^i = \frac{1}{2} + \frac{(q_i - q_j)(\alpha_a - \frac{\gamma_i+\gamma_j}{2})(K-k) + t_r\left(p_j - p_i + \frac{K+k}{2}(\gamma_j - \gamma_i)\right)}{2\Delta} \tag{OA.51}$$

$$n_r^i = \frac{1}{2} + \frac{t_a(q_i - q_j) + \alpha_r\left(p_j - p_i + \frac{K+k}{2}(\gamma_j - \gamma_i)\right)}{2\Delta} \tag{OA.52}$$

, with $\Delta = t_a t_r - \alpha_r(\alpha_a - \frac{\gamma_i+\gamma_j}{2})(K-k)$. Profit of platform i is:

$$\Pi_i(p_i, \gamma_i, q_i, p_j, \gamma_j, q_j) = (p_i + \gamma_i(Kn_r^i + kn_r^j))n_a^i + (f+F)n_r^i k - Fk - cq_i^2 \tag{OA.53}$$

, which we can also express in terms of the gross utility left to A-users: $\tilde{u}_a^i = (\alpha_a - \gamma_i)(Kn_r^i + kn_r^j) - p_i$:

$$\Pi_i(\tilde{u}_a^i, q_i, p_j, \gamma_j, q_j) = \left(\alpha_a n_r^i(K-k) + \alpha_a k - \tilde{u}_a^i\right) n_a^i + (f+F)n_r^i k - Fk - cq_i^2 \tag{OA.54}$$

Expressing the demand functions as a function of quality and type-A utility, we derive:

$$n_a^i = \frac{1}{2} + \frac{u_i - u_j}{2t_a}$$
$$n_r^i = \frac{1}{2} + \frac{q_i - q_j}{2t_r} + \alpha_r \frac{u_i - u_j}{2t_r t_a}$$

We always have that $\frac{\partial^2 \Pi_i}{\partial q_i^2} = -2c < 0$. $\frac{\partial^2 \Pi_i}{\partial u_i^2} = \frac{1}{2t_a t_r^2}(\alpha_r \alpha_a (K-k) - 2t_r t_a)$, is negative if A2 holds. Tedious calculations show that the determinant of second derivatives is positive as soon as A3 holds. Hence under this condition, the first-order approach is valid. First-order conditions of (OA.53) yield:

$$p_i(\gamma_i, \gamma_j) = t_a - \frac{\alpha_r(\alpha_a - \frac{\gamma_i + \gamma_j}{2})(K-k)}{t_r} - \frac{\gamma_i}{2}(K+k) - \frac{\alpha_r}{2t_r}\gamma_i(K-k) - (f+F)\frac{\alpha_r}{t_r}k \tag{OA.55}$$

$$q_i(\gamma_i, \gamma_j) = \frac{\alpha_a - \frac{\gamma_i + \gamma_j}{2}}{4ct_r}(K-k) + \gamma_i \frac{K-k}{8ct_r} + k\frac{f+F}{4ct_r} \tag{OA.56}$$

Given that A-users individual rationality constrains γ_i to be smaller than α_a, we find again that references decrease quality. This effect is mitigated by high reference fees. Symmetry allows to simplify the expressions above to:

$$p_i(k, \gamma_0) = p_i(k) - \gamma_0 \frac{K+k}{2} \tag{OA.57}$$
$$q_i(k, \gamma_0) = q_i(k) - \gamma_0 \frac{K-k}{4ct_r} \tag{OA.58}$$

, with $\gamma_0 = \gamma_i = \gamma_j$ and $q_i(k)$ and $p_i(k)$ given in (9) and (10). Transaction fees have no impact on the overall payment made by A-users, but decreases quality available to R-users. In turn, platform profits increase as γ_0 increase. If $\gamma_0 = \gamma_a$ and $f = F = 0$, platforms replicate the collusive outcome where quality is set at 0, and all surplus is extracted from A-users with 0 participation fees and maximal interaction fees.

OA.2.4 Proofs of Appendix OA.2

OA.2.4.1 Proof of Proposition 4

Assume 1 wants to deter any possible entry strategy of 2. His problem is formalized as follows:

$$
\begin{aligned}
\max_{p_1, q_1} \quad & p_1 - c_1 q_1^2 \\
s.t. \quad & 0 \leq q_1 && \text{(Q)} \\
& p_1 \leq \bar{u}_a + \alpha_a K && \text{(P)} \\
& p_1 - \alpha_a(K-k) - c_2 max((q_1 - \alpha_r)^2, 0) - \delta F k \leq 0 && \text{(A)} \\
& (1-\delta) min(\bar{p}, (p_1 + \alpha_a(K-k)))) - c_2(q_1+\alpha_r)^2 + \delta f k \leq 0 && \text{(R)} \\
& p_1 - \alpha_a(K-k) - c_2(q_1+\alpha_r)^2 + \delta f k \leq 0 && \text{(AR)}
\end{aligned}
$$

Constraint (Q) means quality cannot be negative and (P) constrains price to be lower than A-users willingness to pay for interaction. Constraints (A), (R) and (AR) means no entry strategy is profitable. The Lagrangian writes:

$$\mathcal{L}(p_1, q_1, \lambda) = p_1 - c_1 q_1^2 - \lambda_Q Q - \lambda_P P - \lambda_A A - \lambda_R R - \lambda_{AR} AR$$

Assuming that no constraint is binding ($\lambda \equiv (\lambda_Q, \lambda_P, \lambda_A, \lambda_R, \lambda_{AR}) = 0$) would result in $p_1 = +\infty$ and $q_1 = 0$ thereby breaching constraints (Q and R). As we assume δ is small, it is straightforward that if (AR) is binding , then (R) also is. Thus (AR) can be dropped from the analysis. We first neglect (Q) and (P). We will then show that indeed the candidate solution satisfies these conditions. Our problem therefore reduces to:

$$\mathcal{L}(p_1, q_1, \lambda) = p_1 - c_1 q_1^2 - \lambda_A A - \lambda_R R$$

Again, assuming that $\lambda_A = \lambda_R = 0$ is inconsistent, as a profit maximizer would increase p_1 until either of (A) or (R) is binding. Assume now that $\lambda_R = 0$ and $\lambda_A > 0$. This means only (A) is binding. Inserting p_1 into the objective function the problem of platform 1 becomes:

$$\underset{q_1}{max} \qquad \alpha_a(K-k) + \delta Fk + c_2(q_1 - \alpha_r)^2 - c_1 q_1^2$$

We easily derive that platform 1 therefore decreases q_1, until (R) becomes binding too.

Alternatively assume that $\lambda_R > 0$ and $\lambda_A = 0$. This means only (R) is binding. Plugging p_1 into the objective function the problem of platform 1 becomes:

$$\underset{q_1}{max} \qquad -\alpha_a(K-k) + \frac{\delta}{1-\delta} fk + \frac{c_2}{1-\delta}(q_1 + \alpha_r)^2 - c_1 q_1^2$$

In the relevant case in which $c_2 < (1-\delta)c_1$, it results that platform 1 would optimally set $q_1 = \frac{c_2 \alpha_r}{c_1(1-\delta) - c_2}$. This, however, violates constraint (A) as soon as c_1 an c_2 are not too different, ie if $c_2 \geq \frac{\alpha_a(K-k)}{2\alpha_r^2}\left(\frac{c_1}{c_2} - 1\right)$.

Hence we showed that is has to be that both constraints are binding. (A) met with equality gives an expression for p_1 which we can insert in the objective function and (R) to obtain Proposition 4. We observe that (OA.47) together with assumption A4 ensures that $q_1 \geq \alpha_r$ meaning (Q) is never binding. Similarly, we can show that if c_2 is not too small ($c_2 > \frac{(\alpha_a(K-k))^2}{4\alpha_r^2(\bar{u}_a + k\alpha_a)}$) (P) is never binding either. This completes the proof.

OA.2.4.2 Proof of Corollary 1

We want to prove that the limit cost c_1 below which there is no entry is increasing in k. For this we need to find the cost c_1 that makes platform 1 indifferent between deterring and accommodating entry. We need to find c_1 such that $\pi_1^*(NE) = max(0, \pi_1^*(A), \pi_1^*(R)$. We observed in Section OA.2.2.2 that accommodation by A is always dominated by accommodation by R. Further, $\pi_1^*(R)$ is nonnegative. Hence we need to find c_1 such that:

$$\pi_1^*(NE) = \pi_1^*(R) \tag{OA.59}$$

we assume δ is small, hence c_{lim} solves:

$$G(c_1) \equiv \alpha_a(K-k) + c_2(\tilde{q}_1 - \alpha_r)^2 - c_1(\tilde{q}_1)^2 + \delta F k - \delta(\bar{u}_a + \alpha_a K - Fk) = 0 \tag{OA.60}$$

, with $\tilde{q}_1$ implicitly defined by (OA.47). When $\delta = 0$ it is easy to show that

$$c_{lim} = c_2 \left(1 + \left(\frac{2c_2\alpha_r^2}{\alpha_a(K-k)}\right)^2\right)$$

We observe that c_{lim} increases in k, which means references impede entry. Even when $k = 0$ we have that $c_{lim} > c_2$, which means a platform benefiting from favorable belief may retain its monopoly over a market as long as its cost disadvantage is not too large.

In the general case in which $\delta \geq 0$ we first estimate the impact of k on equilibrium quality when there is deterrence. Applying the implicit function theorem to (OA.47) we derive :

$$\frac{\partial \tilde{q}_1}{\partial k} = \frac{-2\alpha_a + \delta\left(\frac{f}{1-\delta} + F\right)}{2c_2\left(\frac{\tilde{q}_1 + \alpha_r}{1-\delta} - (\tilde{q}_1 - \alpha_r)\right)} < 0 \tag{OA.61}$$

Like in the case with differentiated platforms, quality decreases with references and fees mitigate this effect. We now turn to the effect of references on c_{lim}. We use again the implicit function theorem on (OA.60):

$$\frac{\partial c_{lim}}{\partial k} = \frac{-(\alpha_a - 2\delta F) - 2((c_1 - c_2)\tilde{q}_1 + c_2\alpha_r)\frac{\partial \tilde{q}_1}{\partial k}}{\tilde{q}_1^2}$$

using (OA.61 and focusing on the case when δ is close to 0:)

$$=-2\frac{\partial \tilde{q}_1}{\partial k}\frac{c_1-c_2}{\tilde{q}_1}+o(\delta)$$

Recalling (OA.61), we observe that references increase the limit production cost of 1 below which entry of a competing platform is deterred. References therefore make entry less likely.

OA.2.4.3 Proof of Corollary 2

Corollary 1 showed that, in our setting, references impede entry. We now question whether go-between fees facilitate or complicate entry. First, we investigate the effect of the publisher fee f. We use again the implicit function theorem on (OA.60).

$$\begin{aligned}\frac{\partial c_{lim}}{\partial f} =& -2\frac{1}{\tilde{q}_1^2}((c_1-c_2)\tilde{q}_1+c_2\alpha_r)\frac{\partial \tilde{q}_1}{\partial f}\\ =& -\frac{1}{\tilde{q}_1^2}((c_1-c_2)\tilde{q}_1+c_2\alpha_r)\frac{k\frac{\delta}{1-\delta}}{c_2\left(\frac{\tilde{q}_1+\alpha_r}{1-\delta}-(\tilde{q}_1-\alpha_r)\right)}<0\end{aligned}$$

Hence, a large publisher fee f decreases c_{lim}, meaning entry is facilitated. We now investigate the effect of the sponsor fee F.

$$\begin{aligned}\frac{\partial c_{lim}}{\partial F} =& \frac{1}{\tilde{q}_1^2}\left(2\delta k-2((c_1-c_2)\tilde{q}_1+c_2\alpha_r)\frac{\partial \tilde{q}_1}{\partial F}\right)\\ =& \frac{\delta k}{\tilde{q}_1^2}\left(2-\frac{((c_1-c_2)\tilde{q}_1+c_2\alpha_r)}{c_2\left(\frac{\tilde{q}_1+\alpha_r}{1-\delta}-(\tilde{q}_1-\alpha_r)\right)}\right)\\ \geq& \frac{\delta k}{\tilde{q}_1^2}\left(2-\frac{(c_1-c_2)\tilde{q}_1+c_2\alpha_r}{2c_2\alpha_r}\right)\\ =& \frac{\delta k}{2\tilde{q}_1^2}\left(2-\frac{(c_1-c_2)\tilde{q}_1}{c_2\alpha_r}\right)\end{aligned}$$

When costs are not too different, this expression is positive and publisher fees impede entry. Finally we question whether, given a fixed margin $m = F - f$, high fees favor or discourage entry. In other word, if m is fixed, which of the two effects (quality increase with f, quality decrease with F) prevails. To do so, we vary F and fix f to $f = F - m$:

$$\begin{aligned}\frac{\partial c_{lim}}{\partial F} =& \frac{1}{\tilde{q}_1^2}\left(2\delta k - 2((c_1 - c_2)\tilde{q}_1 + c_2\alpha_r)\frac{\partial \tilde{q}_1}{\partial F}\right)\\ =& \frac{\delta k}{\tilde{q}_1^2}\left(2 - \frac{(c_1 - c_2)\tilde{q}_1 + c_2\alpha_r}{c_2\left(\frac{\tilde{q}_1+\alpha_r}{1-\delta} - (\tilde{q}_1 - \alpha_r)\right)}\frac{2-\delta}{1-\delta}\right)\\ =& \frac{\delta k}{\tilde{q}_1^2}\left(2 - \frac{((c_1 - c_2)\tilde{q}_1 + c_2\alpha_r)(2-\delta)}{2c_2\alpha_r + \delta c_2(\tilde{q}_1 - \alpha_r)}\right)\end{aligned}$$

When $c_1 > c_2$, this expression is negative if α_r is large enough. Hence, keeping the go-betweens' margin constant and assuming δ is arbitrarily close to 0, high fees induce more entry if and only if $\alpha_r > \sqrt{\frac{\alpha_a(K-k)}{2c_2}}$. Otherwise, this conclusion is reversed and small fees are preferable.

OA.2.4.4 Single-homing advertisers

We generalize the setting of Appendix OA.2.1 to non symmetric costs. Each platform i has cost parameter $c_i > 0$. Denote for concision $\Delta(k) \equiv t_r t_a - \alpha_r\alpha_a(K-k)$

Starting from profit function (3), we can re-write the equilibrium quality and prices as:

$$2c_i q_i = \alpha_a(K-k)\frac{p_i}{2\Delta(k)} + (f+F)k\frac{t_a}{2\Delta(k)} \tag{OA.62}$$

$$p_i\frac{t_r}{2\Delta(k)} = -\frac{\alpha_r}{2\Delta(k)}(f+F)k + \left(\frac{1}{2} + \frac{\alpha_a(K-k)(q_i - q_j) + t_r(p_j - p_i)}{2\Delta(k)}\right) \tag{OA.63}$$

Inserting (OA.62) into (OA.63), we find that

$$p_i + p_j = \frac{2\Delta(k)}{t_r} - (f+F)k\frac{2\alpha_r}{t_r}$$

The sum of prices posted by platforms (and hence at least one of these prices) increase with k, when fees f and F are small. Taking fees as high as possible reduces or even may reverse this effect. Unravelling further the calculations, we find that

$$p_i = \frac{1}{t_r} \frac{2\Delta(k) - (f+F)k2\alpha_r}{1 + \frac{\frac{(\alpha_a(K-k))^2}{2\Delta(k)c_i} - t_r}{\frac{(\alpha_a(K-k))^2}{2\Delta(k)c_j} - t_r}} \tag{OA.64}$$

From (OA.64) and (OA.62) we find that in accordance with intuition price increases and quality decreases as c_i increase. Using the envelope theorem on each platform's objective function (3) we have that:

$$\frac{\partial \Pi_i}{\partial c_i} = -q_i^2 < 0$$

When fees are high, the conclusions are reversed.

OA.3 Specialized A-users

We have seen that interplatform references induce a trade-off between providing content diversity to users and decreasing the competition between platforms. On top of that, notice that references operate a sharing of R-users from a platform to the other. This not only increases the number of visits to each outlet, but also alters the average composition of this readership. Indeed, in the presence of interplatform references, the average viewer becomes less specialized, which may alter the decision of platforms regarding which type of firms to contract with.

In this section, we introduce two new types of A-users, in addition to the general A-users described in Section 3. These are specialized firms, who value readership depending on the position of R-users on the Hotelling line. There are two types of specialized firms, 1 and 2. An advertiser of type i is primarily interested in reaching the users of platform

i, possibly because these users have specific characteristics or interests that correspond to the advertising target. The willingness to pay of firms 1 and 2 for each interaction with a user located at distance x from 1 and $(1-x)$ from 2 is:

$$\text{firm 1: } \; WTP_1(x) = \bar{\alpha}_a - \eta_a x$$
$$\text{firm 2: } \; WTP_2(x) = \bar{\alpha}_a - \eta_a (1-x)\,,$$

where $\bar{\alpha}_a$ represents the willingness to pay for a user in the core target (i.e. $x = 0$, for firm 1 displaying ads in platform 1). This willingness to pay is discounted by η_a per unit distance. η_a is positive, meaning that users further away from the core target are less valuable to firms.[3] This specification means A-users do not have a preference for a platform, but for the type of users they gather. We assume that $\bar{\alpha}_a > \eta_a$ such that firms always obtain some benefits from reaching any user. Additionally, users endure a disutility $\eta_r x$ (resp. $\eta_r(1-x)$) from advertisement when they interact with A-users of type 1 (resp. type 2). In the media application, this represents the disutility from specialized advertisement, that increases as the advertising becomes ill-targeted (i.e. x tends to 1/2), capturing the fact that interactions can become less useful to R-users when advertising is not tailored to their needs and interests. The model with specialized firms is illustrated in Figure 3.

[Figure 3 about here.]

If platform contract with specialized firms, the utility of users of news outlet 1 is edited from (1) to:

$$U_r^i = \bar{u}_r + (q_1 - (t+\eta_r) \mid x - x_i \mid - s)K + (q_2 - (t+\eta_r) \mid x - x_j \mid - s)\, k$$

[3]Note that the base case corresponds to $\bar{\alpha}_a = \alpha_a$ and $\eta_a = 0$

This results in demand function

$$n^1 = \frac{1}{2} + \frac{q_1 - q_2}{2(t + \eta_r)}$$

OA.3.1 Conditions for contracting with generalist firms

Assume platforms choose their quality first, and then decide which advertisers to display. Assume further that generalist advertisement yields a constant disutility to all users η_g. We have the following proposition:

Proposition 5 *A platform deviates from general advertisement and chooses specialized advertising over general advertising if and only if:*

$$\alpha_a \leq \frac{(\bar{\alpha}_a - \frac{\eta_a}{4})K + (\bar{\alpha}_a - \frac{3\eta_a}{4})k}{(1 + 2A)K + (1 - 2A)k} \equiv \alpha_a^{lim} \text{ , } \textit{with } A = \frac{\eta_r/2 - \eta_g}{2t + \eta_r} \tag{OA.65}$$

Proof. Once quality has been chosen and its associated costs are sunk, the profits of platform i, when both platforms publish specialized advertisement is:

$$\Pi^i_{Specialized}(q_i, q_j, k) = \frac{1}{2}\left(\bar{\alpha}_a - \frac{\eta_a}{4}\right)K + \frac{1}{2}\left(\bar{\alpha}_a - \frac{3\eta_a}{4}\right)k \tag{OA.66}$$

If platform i switches to general advertisement, the relative disutility from advertising results in a gain of anchored users $A = \frac{\eta_r/2 - \eta_g}{2t + \eta_r}$, and extracts value α_a from them. Hence, profits are :

$$\Pi^i_{General}(q_i, q_j, k) = \frac{\alpha_a}{2}\left((1 + 2A)K + (1 - 2A)k\right) \tag{OA.67}$$

Comparing profits (OA.66) and (OA.67) yields that platform i chooses specialized A-users if condition (OA.65) is met. This condition leads to interesting strategic interactions. The condition for platforms' joint profits to be greater when both platforms choose specialized A instead of both choosing generalist A-users is

$$\alpha_a \leq \frac{1}{K+k}\left(\left(\bar{\alpha}_a - \frac{\eta_a}{4}\right)K + \left(\bar{\alpha}_a - \frac{3\eta_a}{4}\right)k\right) \tag{OA.68}$$

If $A = 0$, (OA.65) and (OA.68) are equivalent: platforms' individual and joint incentives are aligned.

If $A > 0$, (OA.65) implies (OA.68). If (OA.68) is met but (OA.65) is not, a coordination issue prevents platforms to reach the profit-maximizing equilibrium.

If $A < 0$, (OA.68) implies (OA.65). If (OA.65) is met but (OA.68) is not, a prisoner's dilemma arises. ■

A can be positive or negative, depending on whether the nuisance of a general piece of advertisement η_g, is smaller or greater than the nuisance exerted by specialized advertisement on the median user $\eta_r/2$. We observe that the threshold for one platform to deviate to specialized advertising increases with k, meaning general advertisement is more likely to be selected when there are more interplatform references. If one platform switches to specialized advertisement, it is easy to show that the competitor is likely to follow suit. Figure 4, shows the limit α_a^{lim} above which one platform selects general advertisement (solid lines) and the second one follows suit (dotted lines), as a function of the general advertisement externality η_g.

[Figure 4 about here.]

Thus, when the nuisance that specialized A-users yield on marginal R-users is not too small relative to that of specialized A-users, the region in which a platform switches to specialized advertisement decreases with k. This means interplatform references tend to make it more likely that platforms switch to general advertising, as opposed to specialized advertising.

OA.3.2 Quality equilibrium

We saw that the platforms may switch to a paradigm where both choose specialized A-users. Platforms' profit maximization results in quality:

$$q(k) = \frac{1}{4c(t_r + \eta_r)} \left((\bar{\alpha}_a - \frac{\eta_a}{2})K + \left(F + f - (\bar{\alpha}_a - \frac{\eta_a}{2}) \right) k \right)$$

First of all, specialized advertisement helps platforms differentiate from one another, since the transport cost, or disutility from imperfect preference matching t_r is virtually increased to $t_r + \eta_r$. This tends to decrease competition, and therefore leads to a decrease in quality. As before, the exchange fees may further decrease or increase the quality provided to users. The main insights remain: conditional on platforms displaying specialized advertising, interplatform references tend to decrease quality. High reference fees alleviate this effect by restoring competition.

OA.3.3 Surplus considerations

Again, the users most negatively affected by the fact platforms may choose generalist, instead of specialized A-users, are the ones with strong preferences ($x = 0$ and $x = 1$). On top of the effect of the greater preference costs identified in the body of the paper, these users also have to endure a stronger disutility of interaction with A-users. Users with weak preferences lose relatively less utility, or may even gain from interacting with generalist A-users, when $A > 0$. This phenomenon adds to the phenomenon we observed in Section OA.3.1, namely that users with weak preferences are the main beneficiaries of interplatform referencing.

As a conclusion, if a large share of readership originates from interplatform references, platforms may switch to general advertisement, as opposed to specialized advertisement. On top of the effects described in the previous sections, competition between platforms is increased, which benefits both users and social surplus. However, such general advertisement accentuates the discrepancy in utility gains between non-specialized and specialized users.

OA.4 Effect of references on the production of original articles

The body of the paper considers that the quantity of content is exogenous and fixed. It allows for us to focus on quality. However, policy makers may also be concerned about the amount of content created. In fact, quantity can also be seen as a measure of the quality of a platform (Cage, 2017; Berry and Waldfogel, 2010). It is legitimate to fear that interplatform references may reduce the content diversity available to users. Indeed, if new content is shared with the users of a competitor, incentives to produce original content may decrease.

We now assume that quality is symmetric and fixed at $Q > 0$ and we seek to endogenize the quantities produced by each platform. Total quantity produced by platform i is now denoted $K_i = \tau_i N$. We assume that platforms can invest to increase the efficiency τ_i of their technology. Reaching an efficiency of τ_i costs $\beta N^2 \tau_i^2$. As a consequence, producing K_i articles comes at cost βK_i^2. When a new unit of content is produced by i, we need to know whether this content is unique, or common with the other platform. For this we follow a process of content production similar to Anderson et al. (2017) and Calzada and Tselekounis (2018). Each piece of content is randomly drawn among a set of N potential pieces of content. This set is common to both platforms. Hence we have that

$$\begin{aligned} k_i =& \tau_i(1-\tau_j)N \\ =& K_i\left(1-\frac{K_j}{N}\right) \end{aligned} \tag{OA.69}$$

We also assume that only a symmetric share $\phi \in [0,1]$ of unique content is sponsored. The utility of R-users now takes into account that each platform may produce a different amount of content.

$$U^1 = \bar{u}_r + (Q - t_r x)K_1 + (Q - t_r(1-x))\,\phi k_2$$

The indifferent user gives us the demand function:

$$n_r^i = \frac{Q(K_i - K_j) + \phi(k_j - k_i) + t_r(K_j - \phi k_j)}{t_r\left(K_i + K_j - \phi(k_i + k_j)\right)}$$

expliciting k_i as in (OA.69) yields

$$= \frac{Q(K_i - K_j)(1-\phi) + t_r K_j(1 - \phi + \frac{K_i}{N})}{t_r\left((K_i + K_j)(1-\phi) + 2\phi\frac{K_i K_j}{N}\right)}$$

Profits are similar to equation (3), except that quality is now exogenous and platforms compete in the number of articles they produce. Inserting the expression of k_i:

$$\Pi_i(k) = \left(\alpha_a K_i + \phi f K_j\left(1 - \frac{K_i}{N}\right)\right) n_r^i + \phi(\alpha_a - F)K_i\left(1 - \frac{K_j}{N}\right)(1 - n_r^i) - \beta K_i^2$$

First-order conditions relative to K_i and K_j, assuming symmetry, provide an implicit definition of K_i:

$$\begin{aligned} FOC(K_i) \equiv & \frac{\alpha_a}{2} - f\phi\frac{K_i}{2N} + (\alpha_a - F)\frac{\phi}{2}\left(1 - \frac{K_j}{N}\right) \\ & + \left(\alpha_a - \phi\left(1 - \frac{K_i}{N}\right)(\alpha_a - (f+F))\right)\frac{\partial n_r^i}{\partial K_i}K_i - 2\beta K_i = 0 \end{aligned}$$

We assume for simplicity that N is large, meaning that all content is unique to both platforms. In particular, this means that n_r^i is a constant of ϕ. It results that:

$$\frac{\partial FOC(K_i)}{\partial \phi} = \frac{1}{2}(\alpha_a - F) - (\alpha_a - (f+F))K_i \frac{\partial n_r^i}{\partial K_i}$$

Using the implicit function theorem, we write

$$\frac{\partial K_i}{\partial \phi} = -\frac{\frac{\partial FOC(K_i)}{\partial \phi}}{\frac{\partial FOC(K_i)}{\partial K_i}}$$

Noting that the second order derivatives of $FOC(K_i)$ with respect to K_i need to be negative for K_i to be an optimal choice, we derive the sign of $\frac{\partial K_i}{\partial \phi}$:

$$sign\left(\frac{\partial K_i}{\partial \phi}\right) = sign\left(\frac{\partial FOC(K_i)}{\partial \phi}\right) = sign(-(\alpha_a - F)(2Q - 3t_r) + f(2Q - t_r))$$

Under Assumption A1 that ensures that $Q > t_r$, we observe that if fees are sufficiently high, references trigger more article creations. If both fees and equilibrium quality are low, references result in a decrease in content creation. The case when N is small is left for future research.

OA.5 Crowding the clickbait out

In the media industry the go-betweens, named content discovery platforms, do not only direct to high quality content. In fact, they have been widely criticized for circulating clickbait advertising.[4] Clickbaits are links that provide just enough information to tease the curiosity of readers and induce them to click. The landing page is usually of low editorial quality and readers might find themselves interacting with scammers or unverified information. When the go-between chooses which content to display, it needs to trade off the relatively high willingness-to-pay of clickbait producers, with the negative

[4]See https://www.wired.co.uk/article/fake-news-outbrain-taboola-hillary-clinton

reputation effect they may induce, that may turn readers away from their recommended content. Repeated declarations of Outbrain CEO show content discovery platforms take reputation issues very seriously: "our fundamental currency is not a dollar. Our fundamental currency is user trust. If users trust the content, they are gonna come back over and over".[5]

The care for reputation is exacerbated by the increasing usage of ad blockers. These devices block the commercial ads and sponsored content that induce excessive nuisance. Advertisers willing to be whitelisted to meet certain criteria. These criteria include visual characteristics but also the quality and trustworthiness of the source. In our view, these reputation and whitelisting incentives may be why go-betweens do not only refer readers to the lucrative clickbaits, but mingle them high-quality editorial content.

When users set their ad blocker to allow "acceptable ads" (which is by default), they don't see the base service of Outbrain or Taboola. Instead they see a similar service named "Outbrain Smartfeed" or "Taboola Feed". These frames display fewer, better integrated and importantly, higher quality content. We believe this is indicative evidence that content discovery platforms strive to comply with the quality standards of ad blockers.

Readers view the "recommended content" only if they expect that utility derived from these links exceeds their opportunity cost of time of viewing these links. We assume that the opportunity cost of time of viewing any given sponsored link is uniformly distributed on $[0, 1]$, and independent on a reader's location on the Hotelling line. The utility derived from a clickbait, or quality newspaper are normalized to 0 and 1 respectively. A go-between chooses the share Φ of sponsored links that direct to quality journalism. The remaining $1 - \Phi$ are clickbaits. It immediately results that Φ also corresponds to the share of users who visit the recommended content. We assume the revenue per mille (RPM, which corresponds cost-per-click times click-through rate) of clickbaits RPM_c is

[5] See Yaron Galai on https://www.youtube.com/watch?v=lOd0liypSQ4, last accessed on October 31, 2018

greater than the one of quality journalism RPM_q. This means absent considerations on reputation, go-betweens would only serve clickbaits. For simplicity, we neglect the payment of the publisher fee f. The go-between aims at maximizing the following profit function:

$$\Pi(\Phi) = (\Phi RPM_q + (1 - \Phi) RPM_c)\,\Phi$$

It results that a profit-maximizing go-between sets:

$$\Phi^* = \frac{RPM_c}{2(RPM_c - RPM_q)}$$

Hence, even if quality journalism would have a RPM of 0, go-betweens would have an incentive to display some of their links. Indeed, these high quality references allow go-betweens to cater to their reputation so readers do view the recommended content and engage with the lucrative clickbaits. As the intuition suggests, the greater the RPM of quality journalism, the smaller the number of clickbaits.

This observation yields interesting implications for ad blockers. These impose that Φ be greater than a criteria $\bar{\Phi}$ for recommended content to be displayed. If $\bar{\Phi} \leq \Phi^*$ ad blockers have no impact on go-betweens since they always meet the quality criteria in the absence of an ad blocker. If $\bar{\Phi} > \Phi^*$, go-betweens have to increase the amount of quality journalism in their references, which crowd out the clickbaits.

This means users of ad blockers will enjoy a better reader experience, with more quality content being sponsored. However, our results of Section 4 show that quality to all users decreases. As a consequence, ad-blocker users have access to more content, but of a lower quality. Users without an ad blocker are unambiguously harmed since they endure a decrease in quality while they access only $\Phi^* < \bar{\Phi}$ quality content.

Ad blockers are often criticized for decreasing the quality available of the web. The typical argument is that editors see a decrease in revenues due to ads not being displayed.

This dampens their incentives and ability to invest in quality. We unveiled a new effect that also leads to poor quality of content but survives even when ad blockers allow all commercial ads other than low-quality content discovery platforms.

OA.6 Directing users to substitute content

In the body of the paper, we assume that the go-between directs users only to new content the initial platform did not have. This is consistent with our empirical findings (Appendix A) and the affirmed objective of content discovery platforms to induce more traffic, rather than substitute the traffic of publishers towards other content producers. However, in the media sector, the substituability of content is a continuous measure, that is individual-dependent: French readers reading an article on Brexit may consider that another article on the British economy is redundant, while British visitors would not. Similarly, in the case of flight or hotels search engines, it is not clear whether the references induce a substitution, or an expansion of demand.

In this Appendix we assume that the go-between displays γ_u items drawn from the set of k content unique to each platform (as in the body of the paper), and γ_c items drawn from the $K-k$ common content. R-users would, therefore, be interested in viewing all γ_u items original to each platforms. They will however, view the γ_c substitute items in only one platform. Further, we gain realism by assuming that reading one unique article from the competitor may induce a loss of s_u views to the anchor platforms, with $s_u \in [0, 1]$. This takes account of the fact R-users may be time constrained, and reading one more article elsewhere may divert them from reading fully the content of their platform of first choice.

The users make a decision in two stages. First, they decide with which platform to anchor. They join the platform that maximizes:

$$u_r^{i,i}(x,k) = \bar{u}_r - s + (q_i - t_r \mid x - x_i \mid) (K - \gamma_c - s_u\gamma_u) + (q_j - t_r \mid x - x_j \mid) \gamma_u \tag{OA.70}$$

Searching for the indifferent user yields the same demand function as in the main text:

$$n_r^i = \frac{1}{2} + \frac{q_i - q_j}{2t_r} \tag{8}$$

This first choice being made, they view the content of their anchor platform, and the γ_u unique content of their platform of second choice. Then, they are faced with another choice: they view the common content only once, and need to choose on which platform to view it. A possible reason why R-users may want to leave their anchor platform is that all users may not be located at the same distance of each platform, for each piece of content: a reader may like newspaper A more than B for articles about politics and general news, but prefer B when it comes to economics. References allows for them to change to the platform they believe provides the overlapping content in a way that fits their preferences best. Note also that roaming to the other platform may induce a cognitive cost, or additional effort. The quality gap would have to exceed this cost. To account for the heterogeneity in R-users preferences relative to the common content and their cost of switching, we assume that a share $\Gamma_i(q_i, q_j)$ of R-users are "stayers" who view the common content in their anchor i. The other $1 - \Gamma_i(q_i, q_j)$ are "leavers" and view the common content of their platform of second choice. It is natural to make the following assumptions on $\Gamma_i(.)$:

- With symmetric quality in i and j, there are as many leavers in i and j: $\Gamma_i(q, q) = \Gamma_j(q, q)$.
- If quality is symmetric, the number of leavers does not depend on the quality level:

$\Gamma_i(q,q) = \Gamma_0$

- Higher quality in i induces more R-users anchored in i to stay: $\frac{\partial \Gamma_i}{\partial q_i}(q_i, q_j) > 0$
- Higher quality in i induces more R-users anchored in j to leave: $\frac{\partial \Gamma_j}{\partial q_i}(q_i, q_j) < 0$

The profit of platform i is:

$$\begin{aligned}\Pi_i(q_i, q_j) =& \alpha_a n_r^i (K - \gamma_c - s_u \gamma_u) + \left((\alpha_a - F)n_r^j + f n_r^i\right)\gamma_u \\ &+ \gamma_c \left[\left(\alpha_a \Gamma_i(q_i, q_j) + f\left(1 - \Gamma_i(q_i, q_j)\right)\right) n_r^i + (\alpha_a - F)\left(1 - \Gamma_j(q_i, q_j)\right) n_r^j\right] \\ &- c q_i^2 \end{aligned} \tag{OA.71}$$

Note that the profit function of the main text (8) is a simplification of (OA.71) with $\gamma_c = 0$ and $\gamma_u = k$. The term in bracket corresponds to profits generated with the common content that is sponsored in the competitor's pages. It can be decomposed into two terms. The first term is profit α_a generated from anchored stayers, and f generated by anchored leavers. The last term is the profit $\alpha_a - F$ generated by non-anchored leavers.

Profit maximization relative to q_i yields

$$\begin{aligned}2cq_i =& \left(\alpha_a (K - \gamma_c - s_u \gamma_u) - \gamma_u(\alpha_a - f - F)\right) \frac{\partial n_r^i}{\partial q_i} \\ &+ \gamma_c \left(\alpha_a \Gamma_i(q_i, q_j) + f(1 - \Gamma_j(q_i, q_j)) - (\alpha_a - F)(1 - \Gamma_j(q_i, q_j))\right) \frac{\partial n_r^i}{\partial q_i} \\ &+ \gamma_c \left((\alpha_a - f)\frac{\partial \Gamma_i}{\partial q_i}(q_i, q_j) n_r^i - (\alpha_a - F)\frac{\partial \Gamma_j}{\partial q_i}(q_i, q_j) n_r^j\right)\end{aligned}$$

Using the symmetry of the model, we have that in equilibrium $q_i = q_j = q_{eq}$, which induces $n_r^i = n_r^j = \frac{1}{2}$. We derive that

$$\begin{aligned} q_{eq}(\gamma_u, \gamma_c) =& q(\gamma_u) - \frac{s_u \gamma_u \alpha_a}{4ct_r} - \gamma_c (1 - \Gamma_0) \frac{2\alpha_a - f - F}{4ct_r} \\ &+ \gamma_c \frac{\frac{\partial \Gamma_i}{\partial q_i}(\alpha_a - f) - \frac{\partial \Gamma_j}{\partial q_i}(\alpha_a - F)}{4c} \end{aligned} \tag{OA.72}$$

where $q(.)$ is defined in Proposition 2. The first term is the one observed in the main text. The second term is a direct business-sharing effect: references induce R-users to leave their anchor platforms in $\gamma_c(1-\Gamma_0)$ cases. Platforms' returns to attracting R-users decrease accordingly. The third term corresponds to the incentives platforms have to retain their anchored users and induce non-anchored users to roam. It is nonnegative.

Hence, if the referenced content is substitute to the content the publisher produces, then a new *pro*-competitive effect appears: references allow for R-users to compare qualities and revisit their choice, which induces an intensification of competition and an increase in quality. The publisher fee f encourages publishers to let their anchored readers leave. The sponsoring fee F decreases the returns to competing in quality for non-anchored leavers: when content are substitute, reference fees may decrease competition.

Naive sponsors would be willing to pay up to $\bar{F}_c = \alpha_a$ to sponsor some content that is common to both platforms. Naive publishers would be willing to accept to publisher this external content for a fee not lower than $\underline{f}_c = \alpha_a$. When platforms are rational, calculations similar to Appendix OA.1.2 show that the margin $\bar{F}_c - \underline{f}_c$ is smaller with common content than it is with unique content. Hence there is little value creation and a go-between is likely to prefer referring towards unique rather than common content.

We finally note that in the absence of any behavioral bias $\Gamma_i(q_i, q_j) = 1$. Hence equation (OA.72) reduces to the result of Proposition 2 with $k = \gamma_u$ and an additional (negative) term:

$$q_{eq}(\gamma_u, \gamma_c) = q(\gamma_u) - \frac{s_u \gamma_u \alpha_a}{4ct_r} \tag{OA.73}$$

We observe that directing to substitute further decreases equilibrium quality, compared to a situation when links would direct users to independent content. Assuming finally that users always exhaust their time budget constraint ($s_u = 1$), we have that:

$$q_{eq}(\gamma_u, \gamma_c) = q^* - k\frac{2\alpha_a - f - F}{4ct_r} \tag{OA.74}$$

In that case, the only set of feasible fees that induce no loss in quality is $f = F = \alpha_a$. This corresponds to the willingness to accept to publish external content, which coincides with the willingness to pay to sponsor content. With this set of fees, quality, user and social surplus, and platform profit are same as in the absence of references. For bilateral references to strictly increase profit one would need that fees be set below α_a –and therefore below platforms' willingness to accept that their users may click on external links. This would not be individually rational. In our model, the presence of references to substitute content from competitors increases platform's profits only if the reciprocity of references is contractually agreed between both parties: this allows that fees be below α_a. Following the same steps as in Section 5.1 we show that this increase in profit would however come at the expense of quality, user and social surplus.

OA.7 A procompetitive effect of interplatform promotion

So far we have assumed that all users were perfectly informed about the quality of each platform. However, this assumption may overestimate the competitiveness of the market. In reality, R-users are only sporadically informed about the true quality of the platforms they are not used to read. If a platform unilaterally increases its quality, its anchored R-users will be informed of this increase as soon as they visit the platform. Other users won't – if not through advertising or sponsoring. Interplatform promotion, may therefore promote competition because roaming informs users about competitors' quality and existence. This is something the body of the paper ignores, as the setting is one of perfect information. In this section, we aim at capturing the procompetitive effect of information, when platforms are experience goods.

We slightly edit the setting of Section 3 in the following manner. A share μ of R-users is perfectly informed. They therefore behave similarly to the users of the previous sections. However, we add some realism by assuming that their probability to click depends on the perceived quality of the sponsored content. A share $1 - \mu$ have received no information, and are first assigned randomly to a platform. These users actively search for information on competing content, in a move to find their optimal anchorage site. They are therefore more likely to click on the sponsored content. Doing so, they may discover that the sponsoring platform has better quality and decide to anchor there. For simplicity, we assume here that uninformed users are keen to discover new content and view the sponsored content with probability 1.

Since publishers can allow or disallow any content to appear on their pages, they effectively can set the maximum quality of the sponsored content they accept to display. Two effects are at play. On the one hand, publishers want to have high-quality sponsored content, so as to maximize the number of click from its loyal, informed users. On the other hand, they want to make sure that the quality is sufficiently low so that uninformed users are unlikely to find that the sponsored platform is a better place to anchor. This is consistent with our observation that external content is usually relatively old (see Appendix A).

The publisher may accept to publish only sponsored content which quality is lower from the average quality q_j of the competitor (e.g., select older articles, on less relevant topics). Denote v the derating of quality, so that sponsored content has quality vq_j. We assume for simplicity that the publisher makes a take-it-or-leave-it offer to the sponsor, leaving him the opportunity to accept to sponsor this content, or no content at all. This assumption ensures that sponsors always accept the deal, since absent sponsored links they would not attract any roamers at all. The two types of users have different incentives to click on the sponsored content. The μ informed users are not searching for information. They simply click if they expect the sponsored content to be interesting. We assume the choice of whether to click results from a Tullock contest (Tullock, 1980), between clicking, and go to the outside opportunity (e.g., stop reading news and do something else), which yields positive benefit L, standing for Leisure. Hence the probability to click, is $\frac{vq_j}{vq_j+L}$. The $1-\mu$ uninformed users are curious to discover new content, and thus always click. We assume for simplicity that they are naive: they infer that the quality of the content they roam to reflects the average quality of the sponsor platform. Any single click generates gains f to the publisher, and a cost F to the sponsor.

We assume the game is repeated infinitely many times, and platforms have a discount factor $0 < \delta < 1$. When an uninformed user clicks on a sponsored link, the publisher gains f. However, this user might decide to elect the other platform as her anchor site. In that case, the publisher loses a user of future value $\sum_{t=1}^{\infty}(K\alpha_a + fk)\delta^t$, and gains some viewership from sponsored links, coming at cost F : $\sum_{t=1}^{\infty} k(\alpha_a + kf - F)\delta^t$. We assume that $\sum_{t=1}^{\infty}(K\alpha_a + fk - k(\alpha_a - F + kf))\delta^t - f > 0$. This means that platforms prefer to retain uninformed users who would decide, after inspection, to anchor with the competitor. After inspection, an uninformed user chooses to leave platform i and elect the sponsor j as her new anchor site if and only if:

$$q_i - t_r x < vq_j - t_r(1-x)$$

This means the expected number of returning uninformed users is $(1-\mu)\left(\frac{1}{2} + \frac{q_i - v_i q_j}{2t_r}\right)$.

Publishers choose v so as to maximize:

$$\Pi(v) = \overbrace{\mu\left(K\alpha_a + \frac{vq_j}{vq_j + L}kf\right)\frac{1}{1-\delta}}^{\text{informed users}} + (1-\mu)\Bigg(\underbrace{\frac{K\alpha_a + kf}{1-\delta}\left(\frac{1}{2} + \frac{q_i - v_i q_j}{2t_r}\right)}_{\text{uniformed users, staying anchored}}$$

$$+ \underbrace{\left(\left(K\alpha_a + kf + (\alpha_a + kf - F)\frac{\delta}{1-\delta}k\right)\left(\frac{1}{2} - \frac{q_i - v_i q_j}{2t_r}\right)\right)}_{\text{uninformed users, choose rival as anchor}}\Bigg)$$

After some calculations, we find that the optimal v_i^* is explicitly given by:

$$v_i^* q_j = \sqrt{\frac{\mu}{1-\mu}\frac{1-\delta}{\delta}\frac{kf}{((K-k)\alpha_a + Fk - fk(k-1))}2t_r L} - L$$

Hence publishers don't necessarily want to publish the best pieces of work of their competitors, for fear that they may lose some of their users. The chosen quality is higher when the number of informed users is large, and the relative value of anchored to roaming users decreases (i.e. F decreases).

This section has shown that publishers may preferably select content items in the lower range of quality of their competitors. We argue this may be an attempt to limit the permanent leakage of users to competitors, by providing them a biased signal of the quality of competing platforms. This finding may explain why our regressions in Table 2 reveal that sponsored articles are older if they are issued by a competing news outlet, rather than another news outlet of the same press group.

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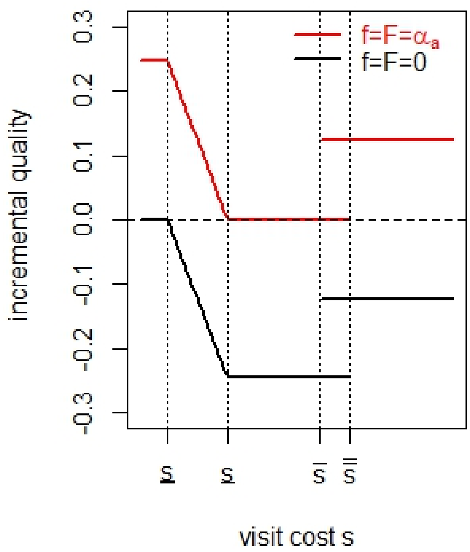
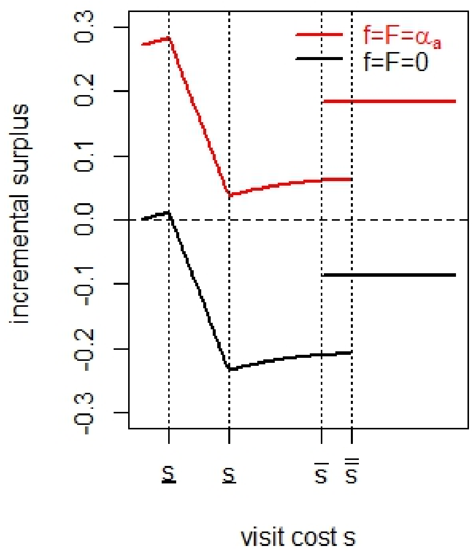
List of Figures

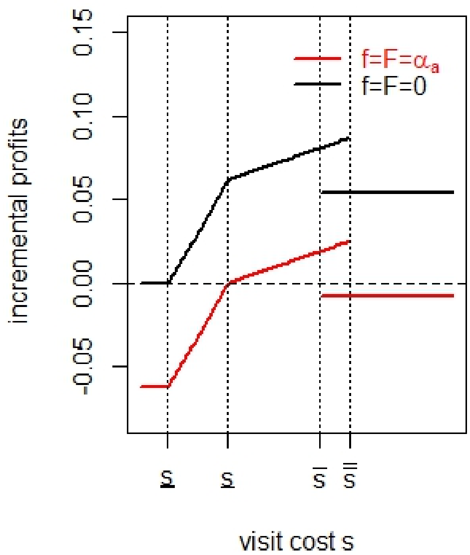
Figure 1: Incremental quality, user surplus, platform profit, and social surplus caused by the presence of a go-between, as a function of the visit cost s. Black lines show the case in which there are no fees. Red lines are high fees $f = F = \alpha_a$.
$t_r = 1, K = 1, k = 0.1, \alpha_a = 0.5, c = 0.1$



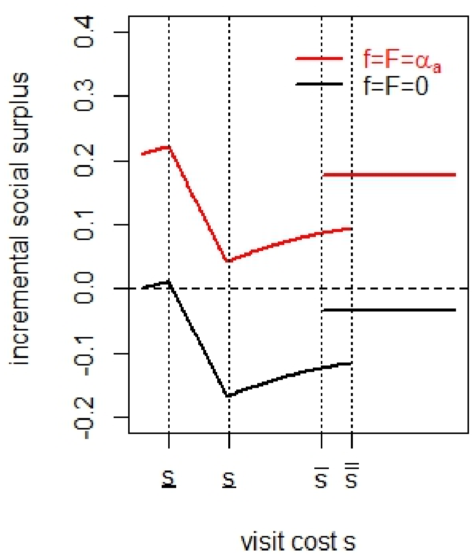
(a) Incremental quality



(b) Incremental user surplus



(c) Incremental platform profits



(d) Incremental social surplus

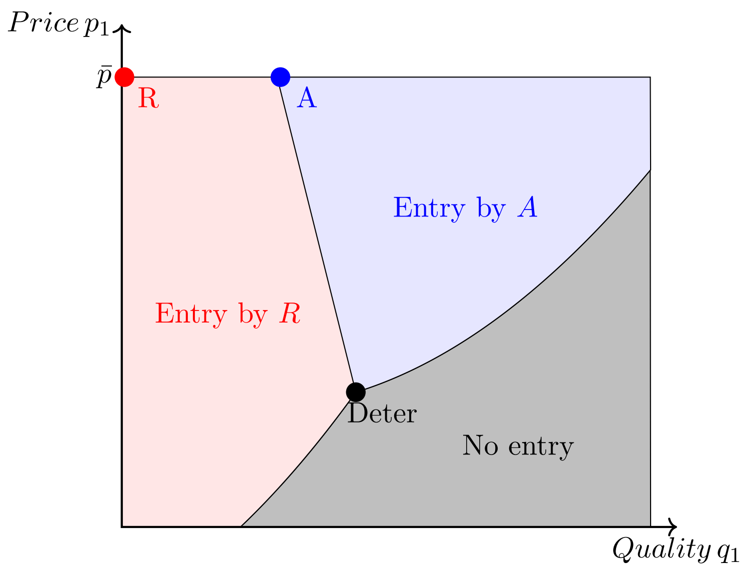


Figure 2: Graphical representation 2's best responses to 1's price p_1 and quality q_1. The black dot is a candidate for 1's best entry deterrence strategy. R (A) dots are 1's best accomodation strategies when 2 enters by R (A)

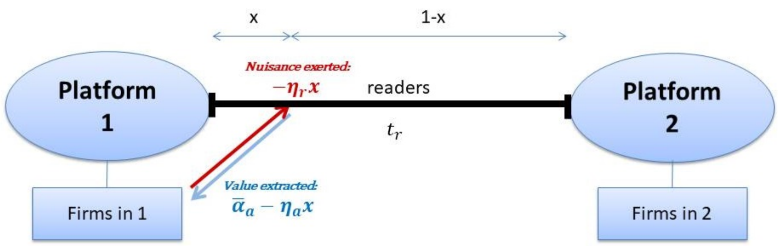


Figure 3: Base model with a go-between and specialized advertisement

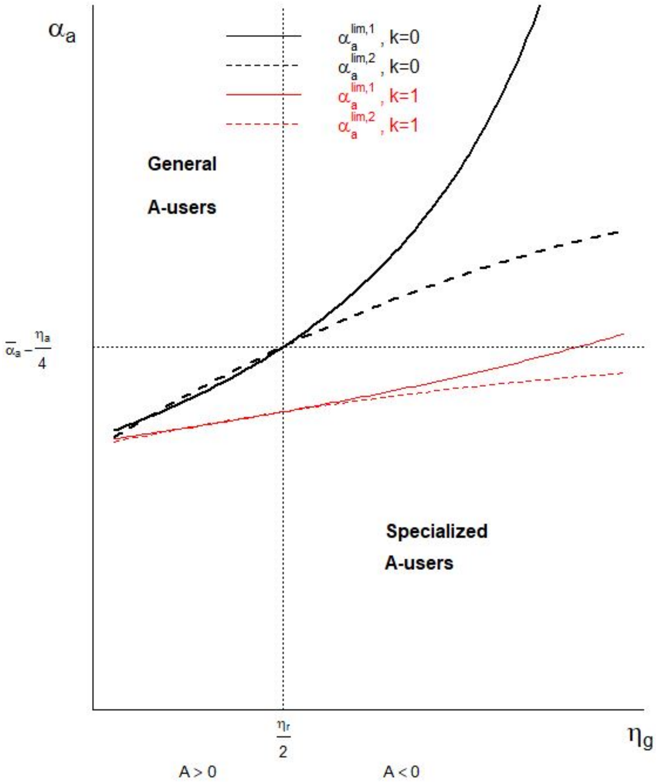


Figure 4: limit α_a^{lim} below which one platform switches to specialized advertisement (solid lines) and the second one follows suit (dotted lines), as a function of the general advertisement externality η_g. Black lines are the benchmark case when there is no inter-newspaper promotion ($k = 0$). Red lines correspond to the case when there is ($k = \frac{K}{3}$).

$\bar{\alpha}_a = 1$, $\eta_a = 0.9$, $K = 2$, $\eta_r = 0.5$, $t = 1$